

## Avoiding Common Pitfalls in Making Pulse Measurements in the Picosecond Domain

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### INTRODUCTION

The purpose of this application note is to discuss measurement problems, with particular emphasis upon those that are unique to the picosecond domain. This note builds upon a previous PSPL application note, AN-3a, from 1988 [1].

For engineers and technicians who have spent their careers in the audio, video, MHz RF, and/or MBit/sec digital world, their first experience in the picosecond world can be very frustrating. Crossing the < 1 ns frontier is the entry into the world of "Distributed Electronics" and microwave techniques. Recall that the reciprocal of 1 ns is 1 GHz, which is the microwave frontier. Analysis and design within an integrated circuit can still be handled as lumped circuit elements for speeds down to perhaps 300 ps. For faster speeds, definitely < 100 ps, and on larger area IC chips, the interconnects between circuit elements, and often the circuit elements themselves, need to be considered as distributed circuits. Even the simple wire bond from an IC chip to its carrier and out onto the pc board must be considered as a distributed transmission line.

For low frequency circuits of up to a few hundreds of MHz and times of a few ns, we could get away with assuming that everything happened simultaneously from one side of a pc board to the other. Our designs and real circuits matched our simple schematic diagrams with lumped resistors, capacitors, inductors, transistors, diodes, etc. We could use a high impedance oscilloscope probe with a 500 MHz bandwidth oscilloscope and be confident we were getting valid measurements.

For high-speed (> 1 GHz, < 1 ns) circuits, this simultaneous time or lumped element assumption is definitely not valid. Recall that the time delay encountered by a light wave (or radio EM wave) is given by:

$$T_d (\text{time delay}) = l (\text{length}) / v_p (\text{prop. velocity}) \quad (1)$$

$$v_p = c / (\epsilon)^{1/2} \quad (2)$$

$c$  is the speed of light ( $3 \times 10^{10}$  cm/sec), and  $\epsilon$  is the relative dielectric constant of the medium. In air, this equates to 33 ps/cm of delay. On a 30 cm, FR-4 pc board ( $\epsilon = 4.7$ ), the delay from board edge to board edge is a whopping 2.2 ns. On a large area silicon IC ( $\epsilon = 11.8$ ), the time delay is 115 ps/cm.

For picosecond measurements, it is no longer possible to "probe" directly the circuit node of interest. All measurements must be considered to have been made at the end of a transmission line and the results interpreted with this in mind. The low frequency concept of "High" impedance of M $\Omega$  or even k $\Omega$  no longer exists. Even free space impedance of 377  $\Omega$  is "High Z" impedance in this domain. No measurement can be considered as "non-intrusive" but will extract power from the circuit under test and affect its performance.

### BANDWIDTH CONSIDERATIONS

To make accurate pulse measurements, it is important to understand and select the appropriate measurement instrument and accessory components, such as cables, attenuators, amplifiers, etc. The first consideration must always be to have an adequate frequency response in the composite measurement system. If you know the approximate risetimes ( $T_r$ ) of the signals to be measured, a good first order approximation to the effective -3 dB bandwidth (BW), of these signals can be obtained from the following equation:

$$T_r(10\%-90\%) * BW(-3dB) \approx 0.35 \quad (3)$$

Ideally all of the measurement system components will have Gaussian responses. Why Gaussian? The Gaussian is the optimum pulse response. See Figures 1 and 2 in PSPL AN-7a, [2] for examples of the Gaussian time domain and frequency domain responses. With the Gaussian wave shape, pulse signals rise and fall smoothly and then rapidly settle to their final values. The presence of other perturbations on a pulse signal, such as precursors, overshoot, and ringing, are all undesirable artifacts. These perturbations extend well beyond the risetime interval and create uncertainty in the actual value of a pulse for a much longer time duration.

With the Gaussian, one knows the final value very rapidly after the risetime. For a pure Gaussian, the risetime \* bandwidth product is 0.332. For realizable, near-Gaussian systems, the 0.35 product given above in equation (3) is closer to reality.

For a cascade of "n", near-Gaussian response components and instruments, the output risetime is given by the root-sum-of-squares of the individual risetimes.

$$Tr(out) = [Tr1^2 + Tr2^2 + \dots + Trn^2]^{1/2} \quad (4)$$

From equation (4), we can thus calculate that if we want to directly measure risetime with 5% or better accuracy, then the composite risetime of our measurement system must have a risetime faster than 1/3 of the signal to be measured. To measure accurately a 100 ps risetime, we thus need a measurement system with a bandwidth in excess of 10 GHz.

Obviously, we oftentimes do not have the necessary bandwidth/risetime available and thus our measurements will be distorted, and we then need to rely upon equation (4) to get an estimate of the true risetime of the signal under test. It then becomes very important to have knowledge of the bandwidth/risetime limitations of our various measurement system components.

## OSCILLOSCOPES

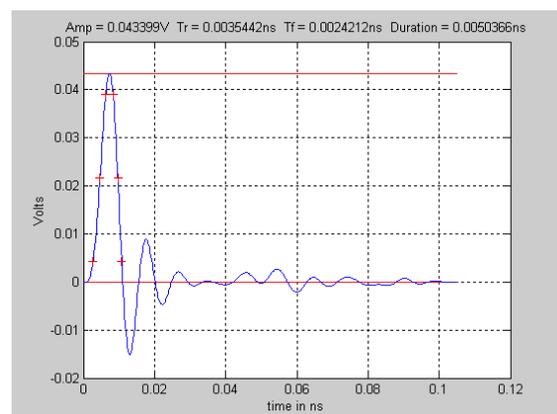
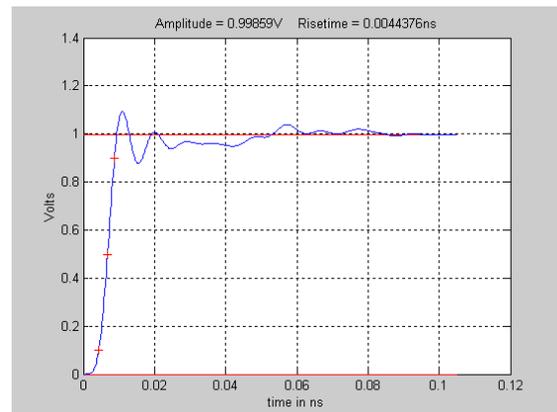
The basic measurement instrument for all time domain waveforms is the oscilloscope. Oscilloscopes are usually specified by their -3 dB bandwidth. Their risetimes can be estimated from equation (3).

Dramatic advances have been made recently in the bandwidth of real-time, digital oscilloscopes. The current leader is LeCroy with their 45 GHz, 10.5 ps, \$268,000 Wave Master, model 845Zi-A. Close behind is Agilent with their 32 GHz, 12.5 ps, \$266,000, Infiniium, model DSOX93204A. In third place is Tektronix with their 20 GHz, 18 ps, \$166,000, model DPO/DSA7204C.

Even higher bandwidths, faster risetimes, and higher A/D bit resolutions are possible using sampling techniques. The major disadvantage of sampling oscilloscopes is that they require a repetitive waveform and cannot be used to measure single transients. The state of the art is presently at 100 GHz bandwidth (4 ps). This is the LeCroy WaveExpert model 100H, using the PSPL/LeCroy

model SE-100 sampling head. (**Note:** PSPL designed and built on an OEM basis the sampling heads for LeCroy.) Close behind are Tektronix with their 70 GHz, 5 ps, model 80E06 and Agilent with their 70 GHz (unspecified risetime) model 86,118A.

The first measurement pitfall is to assume that all oscilloscopes of equal bandwidth will give the same displayed measured waveform. Unfortunately there are significant differences between different manufacturers' oscilloscopes step and impulse responses, even though they may claim the same -3 dB bandwidth. Starting in 1986 through 2001, PSPL published a series of application notes comparing all commercially available sampling oscilloscopes with risetimes of 35 ps or less [3]. The fastest risetime pulse generator then available was used to test the step, transient responses. The long term, nanosecond settling times were also tested. The reader is referred to these application notes to prove to oneself that there are significant waveform differences among oscilloscopes. A future PSPL application note will compare the responses of today's 50 to 100 GHz sampling oscilloscopes.



**Fig. 1** Step and Impulse Responses of PSPL/LeCroy SE-70, 70 GHz, 4.44 ps risetime, calibrated sampling head. PTB calibration [4] for 100 ps time window.

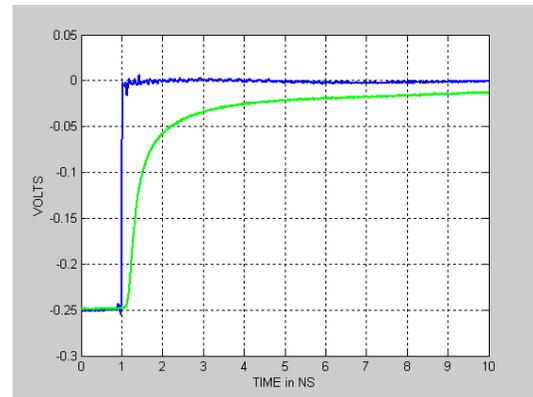
Ideally the user would have available the complete, calibrated step and impulse response of their oscilloscope such as shown in Figure 1. The reality is, however, that oscilloscope manufacturers do not supply such data. They normally only give the typical -3 dB bandwidth. However, such data can be obtained in a formal calibration from national standards labs such as PTB [4], NPL, and NIST.

Today's modern sampling oscilloscopes are quite accurate for both DC voltage and also for their precision time bases. The usual DC voltage error is a thermal drift in the zero volt baseline offset. The vertical gain errors are typically less than the offset error.

There is, however, a potential source of voltage error that is often overlooked. This is the input impedance. All high bandwidth oscilloscopes should have a 50 Ohm input to match the 50 Ohm coaxial cables that are used to connect the signal source to the oscilloscope. The PSPL/LeCroy samplers are specified to be  $50 \Omega \pm 1\%$ , i.e.,  $\pm 0.5 \Omega$ . The Tektronix samplers are specified to be  $50 \Omega \pm 0.5 \Omega$  or  $\pm 1 \Omega$ . The Agilent samplers have much looser specs of only "nominally 50 Ohms" and give no % accuracy statement. Of particular concern is the Agilent model 86118A, 70 GHz sampler, which is specified to have an input reflection coefficient of 20%, which is an input resistance of  $75 \Omega$ . Actual measurements at PSPL have shown the input impedance is in fact resistive at  $71 \Omega$ . Input impedance mismatch causes reflections back into the system under test. If the system is not well back-matched, then these reflections will be re-reflected and appear later in time as distortions on the waveform. For digital data eye diagrams, these mismatch reflections cause closing of the eye diagrams.

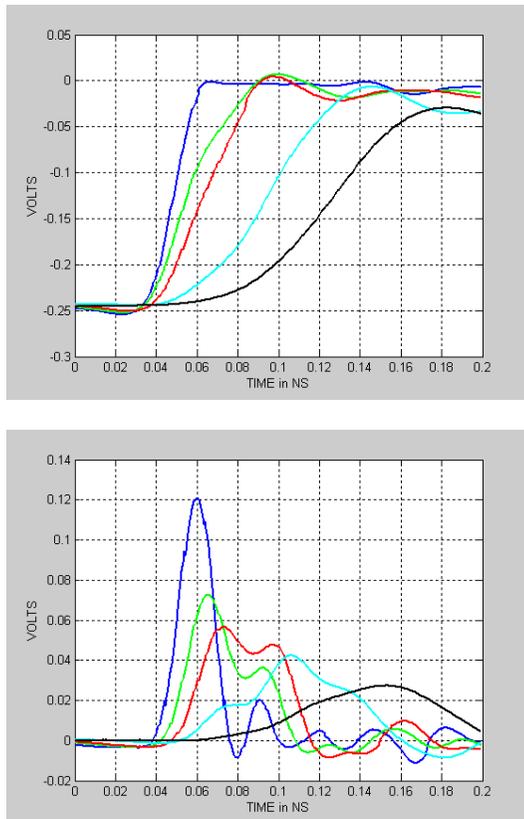
## COAXIAL CABLES

For high bandwidth, picosecond domain measurements, we are always using  $50 \Omega$  coaxial cables to connect our device or signal source under test to our measurement oscilloscope. These cables are the most commonly overlooked pitfall for waveform measurement errors. It comes as a shock to newcomers when they first discover that a coaxial cable can severely distort a pulse waveform.



**Fig. 2** Pulse Response of Coaxial Cable. 20ps rise Input (blue) and Output (green) from 28 ft. of RG-58C/U. 10 ns time window.

An ideal cable would not distort a pulse. However, if any ohmic loss is present in either the conductors and/or dielectric, then waveform distortion will occur. Every coaxial cable is a low pass filter. All cables have losses that rise with frequency. Figure 1 is the pulse response measured when transmitting a 20 ps risetime step pulse through a long length (8.5 m) of ordinary RG-58C/U coaxial cable. This waveform is the typical response of a coaxial cable in which the primary loss mechanism is due to skin effect in the conductors. Wigington and Nahman's classic paper [5] on this subject has shown that this curve is closely approximated by the Complimentary Error Function (CERF), whose argument is inversely proportional to the square root of time. The CERF function has a rapid rate of rise up to the 50% level and then rolls over and takes an extremely long time to move up towards the 100% level. It is extremely difficult to assign a risetime value to a coaxial cable due to this CERF response, which becomes very flat in the 90% region. It should also be noted that the CERF response is most definitely non-Gaussian. Thus, the risetime equation (4) is not valid for cables exhibiting this CERF response.



**Fig. 3** Step and Impulse Responses of RG-58C/U coaxial cables. Input 20ps Step or Impulse (blue), 9" (green), 1 ft. (red), 3 ft. (cyan) and 6 ft. (black) 200 ps time window.

Figure 2 is an extreme example for picosecond measurements. However, even very short lengths of coax can still cause serious waveform errors in the picosecond domain. Cable runs of 1 ft. to 3 ft. are not uncommon in ordinary test set-ups. A very common cable found in all labs is made of ordinary RG-58 coax with either BNC or SMA connectors. Figure 3 shows the distorting effect of passing a 20 ps risetime step pulse or 20 ps duration impulse through short lengths of RG-58C/U. The step leading edge is slowed and the amplitude is decreased. On slower time scales the slow CERF function dribble-up would become apparent. For the impulse responses, note both the smearing in time and the dramatic loss of amplitude with increasingly longer cables. The conclusion to be drawn from Figure 3 is **"NEVER USE RG-58 CABLE for PS MEASUREMENTS !"**

### MICROWAVE QUALITY CABLES

As discussed above, the major loss mechanism in coaxial cables and the cause of waveform distortion is skin effect. Skin effect is the electro-magnetic wave phenomena in which an EM wave's penetration into a conductor follows a decreasing exponential. As the EM wave frequency increases, the depth of penetration decreases. The net effect is to make the effective cross-sectional surface area of a conductor less and the effective resistance increases as a function of frequency.

Consulting the web sites of coaxial cable manufacturers, one will find that they all provide data, either in graphical or tabular form, of the cable loss vs. frequency. A typical plot of loss in dB/unit length vs. frequency on a semi-log scale will show a rising straight line with a slope of 1/2.

Most of the skin effect loss occurs in the smaller cross sectional area center conductor. There are several techniques used by manufacturers to minimize skin effect loss. (1) Increase the cable diameter, (2) silver plate the copper center conductor, and (3) use a lower dielectric constant material for the insulator.

Increasing the cable's outer diameter has a limitation. There is a finite upper frequency limit below which only TEM waves propagate in the cable. Above the cut-off frequency, higher order TE and TM waveguide modes will also propagate in the cable. When this happens, severe waveform distortion occurs to pulse waveforms. For commonly used microwave cable sizes, these limits are listed in Table I.

**Table I** Frequency Limits of Common Microwave, Semi-Rigid Cables  
( $R_0 = 50 \Omega$ , Teflon Insulator, Copper Conductors)

Outer Diameter	Dielectric Diameter	Cut-Off Frequency
0.39"	0.332"	12 GHz
0.25"	0.209"	19 GHz
0.141"	0.1175"	34 GHz
0.085"	0.066"	61 GHz
0.047"	0.037"	109 GHz

Unfortunately, operating at higher frequencies mandates smaller diameter cables with attendant higher dB/m losses. Table II gives typical losses.

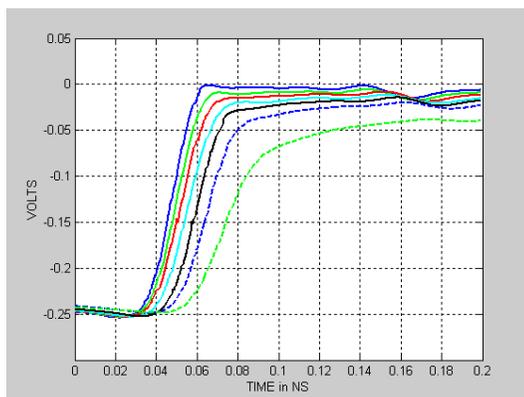
**Table II** Attenuation of Semi-Rigid Cables

Outer Diameter	Insertion Loss in dB/meter	
	1 GHz	10 GHz
0.39"	0.15	0.66
0.25"	0.22	0.89
0.141"	0.37	1.36
0.085"	0.64	2.21
0.047"	1.12	3.73

The most commonly used, low-loss dielectric material for microwave cables is PTFE (Teflon,  $\epsilon = 2.02$ ). By foaming the PTFE, its dielectric constant can be lowered to as low as 1.3, but it becomes very soft and porous then. With a lower dielectric constant, the diameter of the center conductor can be increased, while still maintaining the same diameter outer conductor and impedance. The net result is less skin effect loss in the center conductor.

Another dielectric constant lowering technique used by cable manufacturers, particularly for larger diameter cables, is to support the center conductor with narrow PTFE splines or hollow PTFE tubes. These cables are excellent for narrow band, RF transmission applications. They should not be used for pulse transmission because the non-uniform dielectric structures cause severe phase distortion of pulse waveforms.

Microwave-quality coaxial cable is required for precision, picosecond domain measurements. The most commonly used cable is UT-141-50 with SMA connectors. This is a 0.141" diameter, semi-rigid coax with a solid, silver-plated copper center conductor. The outer conductor is solid copper and the dielectric is Teflon. The 50  $\Omega$  impedance is very uniform and can be held by the manufacturer to less than  $\pm 0.5 \Omega$ . There is also available a semi-flexible version of this cable that has a braided copper outer conductor and nearly identical frequency/pulse response characteristics. Figure 4 shows the pulse responses of several short lengths of 0.141" semi-flex cable when tested with the 20 ps risetime step pulse. While these responses are much improved over the RG-58 cable, Figure 3, they still show risetime slowing and CERF behavior.



**Fig. 4** Step Response of Semi-Flex 0.141" SMA coaxial cables. 20 ps Step Input (blue), 6" (green), 12" (red), 18" (cyan), 1m (black), 4 ft (blue dash), and 8 ft (green dash). 200 ps time window.

There are also available from several manufacturers even higher quality cables with a bit lower loss. These cables may give you lower loss, or improved flexibility, and/or more ruggedness. For the highest bandwidth, fastest risetime measurements, they are mandatory, despite their much higher cost.

When selecting coaxial cables to use, follow these guidelines:

- (1) Determine the highest frequency content in the signals to be measured.
- (2) Based upon the frequency, select the largest diameter cable from Table I.
- (3) Use appropriate coaxial connectors for the cable size and frequency range.
- (4) Always use the shortest possible length of cable to configure your measurement setup.
- (5) Whenever possible, avoid using a coaxial cable at all.

With the LeCroy and Tektronix sampling oscilloscopes, extender cables are available as accessories for their miniature sampling heads. It is better to physically move the sampling head to the signal source than to use a coax cable to transport the signal to the oscilloscope. Unfortunately, this is not possible with the Agilent sampling oscilloscopes. Their sampling modules are very large and must remain in the oscilloscope mainframe. The sole exception is the Agilent model 86118A sampling plug-in, which has the 70 GHz sampler in a small pod at the end of a long umbilical cable.

Another consideration when using coaxial cables is Static Electricity Discharge. Much of the modern, high-speed electronics equipment is susceptible to damage from electro-static discharge. For example, the most common failure mode for expensive sampling heads is exceeding their maximum input ratings of typically only 2 to 3 V. When using coaxial cables, always ask yourself, "Where was this cable last used?" A coaxial cable is an excellent capacitor and can hold a DC voltage for a very long time. Thus, good practice is to always use a metal tool to short the center conductor pin to the outer shell and thus discharge the cable prior to connecting it to any sensitive electronics.

**CONNECTORS**

The most commonly used coaxial connector for microwave and picosecond pulse work is the SMA. The SMA was originally designed for use with 0.141" semi-rigid cable. The original design used the center conductor of the semi-rigid cable as the center pin for the male (plug) connector. Its upper frequency limit is 18 GHz, although there are some enhanced, precision versions that work up to 26 GHz. Most microwave and ps pulse measurement instrumentation uses either the type N, SMA, or higher frequency variations of the SMA. Table III lists these connectors.

**Table III** Common Coaxial Connectors for Microwave and PS Pulse Instrumentation

<u>Name</u>	<u>Diameter</u>	<u>Max. Frequency</u>
N	7 mm	11 GHz (18 GHz)
SMA	4.13 mm	18 GHz (26 GHz)
3.5 mm	3.5 mm	34 GHz
K	2.92 mm	46 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
1 mm	1 mm	110 GHz

The normal type N connectors are rated up to 11 GHz. There are, however, precision type Ns that are rated to 18 GHz. The SMA, 3.5 mm, and K connectors are all compatible and may be mated together interchangeably. The 2.4 mm and V connectors look similar to the SMA, but their internal dimensions are different. The 2.4 mm and V are compatible and may be mated together interchangeably. Do not attempt to mate them with the SMA, 3.5 mm, or K as they will be destroyed in the process. The precision required to machine these connectors (and the price) goes up dramatically as the size decreases and the upper frequency increases.

There are lots of other connector types in use for interconnecting microwave modules, but they typically are not found on test instrumentation and thus are not listed here. When one is testing a module with a different connector, it is then necessary to use an adapter. When purchasing adapters, always buy high quality, name brands. There now are available on the market a lot of cheap knockoffs, which have poor RF design and do not provide the anticipated bandwidth.

A last note relative to connectors: Always remember to properly tighten all connections. A loose connector can lead to over-moding problems and distortion of a pulse waveform. For the SMA-style

connectors, the use of a torque wrench is recommended. Avoid over-torquing a connector as mechanical damage will result.

**ATTENUATORS**

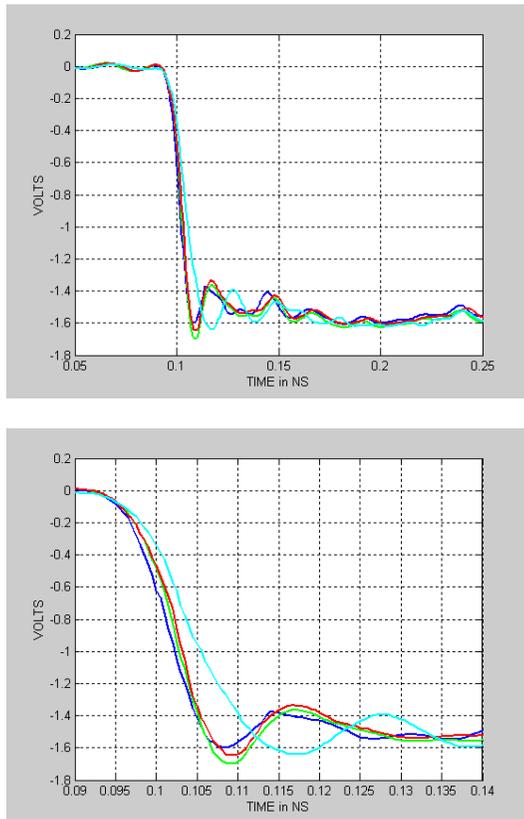
Whenever signal amplitudes exceed the dynamic range of our oscilloscope, we must use attenuators on the input of the scope. It is important that the bandwidth of the attenuator(s) be equal to or greater than the oscilloscope's bandwidth to avoid additional degradation of the measurement quality. The input connector on the oscilloscope will be one of those listed in Table III and will be compatible with the bandwidth of the oscilloscope. Attenuators with the same connector should be used. Most attenuator manufacturers do not specify either the -3 dB bandwidth or the risetime of their attenuators. Their frequency specs are typically those of the mode-free range of the connectors, and they also typically give the attenuation flatness over this range. To properly remove the risetime effects of the attenuator(s) using the RSS equation (4), it is important to know the risetime of each individual attenuator used in a measurement.

PSPL sells attenuators with specified risetimes. See Table IV. The 5510 has SMA connectors while the K and V versions have 2.92mm and 1.85mm connectors.

**Table IV** PSPL Broadband Attenuators

<u>Model Number</u>	<u>Frequency Range</u>	<u>Atten Flatness</u>	<u>Risetime</u>	<u>for Use</u>
5510	DC-18 GHz	±0.6dB	8 ps	>25 ps
5510-K	DC-40 GHz	±1.0dB	5 ps	>15 ps
5510-V	DC-60 GHz	±1.2dB	5 ps	>10 ps

Figure 5 shows the response of these three different attenuators when driven by an ultra-fast, 6 ps falltime, PSPL 4016 step generator.



**Fig. 5** Pulse response of PSPL attenuators. 8 ps falltime Input (blue), 5510V-10dB (green), 5510K-10dB (red) and 5510SMA-10dB (cyan).

Attenuator waveforms scaled by 10dB for ease of comparison. Top plot 200 ps time window. Bottom plot 50 ps time window.

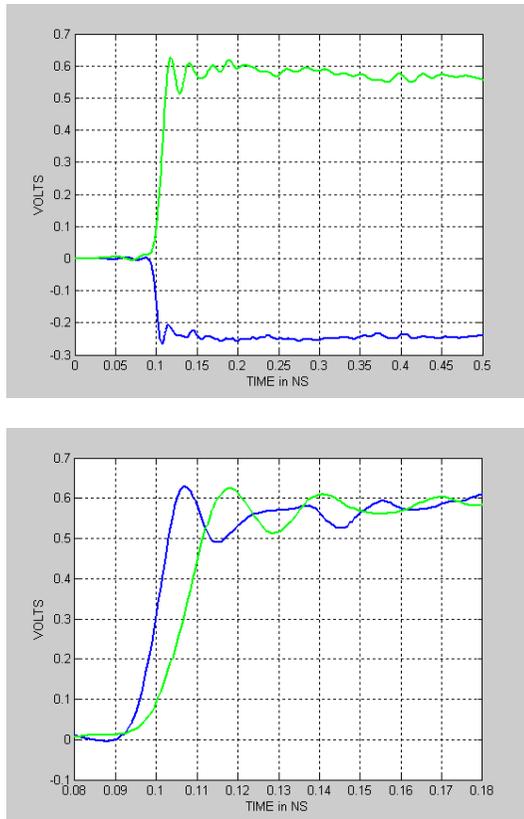
Another often-overlooked measurement pitfall is to not know the precise DC calibration of an attenuator. Too many times, we automatically assume that, if we are using a 20 dB attenuator, the voltage correction factor to be used on the oscilloscope measurement is simply 10 X. Most broadband coaxial attenuators have rather loose specs. A spec of  $\pm 0.5$  dB means a potential voltage measurement error of  $\pm 6\%$ . When calibrating an attenuator for DC insertion loss, use a DVM along with a well-regulated DC power supply plus a precision 50.0  $\Omega$ , series source resistor and a precision 50.0  $\Omega$  load resistor. Most modern digital sampling oscilloscopes will allow the user to enter the exact dB value into the scope to then automatically correct the measured data.

Another source of measurement error is to ignore the input impedance mismatch. Many broadband coaxial attenuators have a typical spec of VSWR  $< 1.2$ . This means an input resistance range from 41.7  $\Omega$  to 60  $\Omega$  and voltage mismatch errors up to  $\pm 9\%$ . It is thus important for precision measurements to hand-select your attenuators to have input resistances as close as possible to 50.0  $\Omega$ . A simple ohm-meter test is sufficient, but remember to always terminate the attenuator output in 50.0  $\Omega$  when performing this test.

### AMPLIFIERS

When we are measuring weak signals, we often find it necessary to add a broadband amplifier on the input of our oscilloscope. The comments above regarding attenuator bandwidth, risetime, insertion loss calibration, and mismatch are equally valid for the selection of amplifiers. Another measurement pitfall with amplifiers is to ignore the effect of AC-coupling and low frequency cutoff when selecting an amplifier. Most amplifiers are not DC-coupled, and thus the low frequency rolloff can cause waveform distortion. For measuring pulse waveforms and eye diagrams, the presence of an AC-coupled component, such as an amplifier, introduces a high pass filter with an attendant sag in waveforms when viewing on long time scales. Many amplifier manufacturers advertise 'broad-band' amplifiers, but they may only cover a couple of decades of bandwidth. Avoid these amplifiers for pulse measurements. Always use amplifiers that have many decades of bandwidth and give low frequency cutoffs of a few MHz or, even better, down to a few kHz.

Amplifiers tend to have poorer frequency responses and pulse responses compared to broadband attenuators. It is thus important to know the actual step response of your amplifier. Figure 6 is an example of the step response of a PSPL ultra-broadband, 20 kHz - 43 GHz, model 5881 amplifier.



**Fig. 6** Pulse Response of PSPL 5881, 8 dB, 20 kHz - 43 GHz Amplifier. 8 ps falltime. Input (blue) and Output (green). Top plot, 500 ps time window. Bottom plot, input inverted and amplitude scaled by 8 dB gain for ease of comparison. 100 ps time window.

### WAVEFORM PLOT NOTES

The 20 ps risetime test setup for the coaxial cable tests, Figures 2, 3, and 4, consisted of a PSPL/ LeCroy ST-20 TDR, 10 ps pulser followed by a PSPL 5935-15GHz low-pass filter. Waveforms measured with PSPL/LeCroy SE-50, 50 GHz sampler.

The 8 ps falltime test setup for the attenuators and amplifier, Figures 5 and 6, consisted of a PSPL model 4016, 6 ps, -5 V pulser, and a 5510V attenuator to create the 8 ps, -1.5 V or -0.25 V test pulse. Waveforms measured with a PSPL/LeCroy SE-100, 100 GHz sampler.

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