Removing Jitter From Picosecond Pulse Measurements

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INTRODUCTION: Uncertainty is always present in every measurement. Uncertainties can be classified as systematic and random. For picosecond pulse measurements, digital sampling oscilloscopes are used. For these oscilloscopes, there are three major uncertainties, namely the impulse response, vertical noise, and timing jitter. The purpose of this application note is to discuss jitter in detail and describe a couple of methods to remove it from oscilloscope measurements.

h(t): The first, and usually dominant, uncertainty is systematic. It is the oscilloscope's transient response, which can be characterized as either its impulse response, h(t), or its step response. The oscilloscope manufacturer typically gives a very limited amount of information about this by specifying only the step response 10%-90% risetime and/or the -3dB bandwidth. With this information, the user can make some educated, "back-of-the-envelope" estimates of the accuracy of his measurements. More precise measurements can be performed if the user knows the exact nature of the oscilloscope's impulse response. National standards labs, such as NIST, NPL, and PTB, provide calibration services to determine an oscilloscope's impulse response. Using deconvolution, the oscilloscope's impulse response waveform distorting effects can be removed from a measurement, and a more accurate picture of the true pulse waveform can be obtained. PSPL's application note, AN-18, discusses the details of deconvolution [1].

NOISE: No matter what the measurement instrument is, the smallest signal it can measure is ultimately limited by noise. Noise is usually a random process and has a symmetric Probability Density Function (PDF). An oscilloscope is basically a time resolved voltmeter. Thus, its ability to measure voltage is ultimately limited by its noise floor. White noise has a uniform distribution for all frequencies. Thus, for an instrument such as a sampling oscilloscope, the wider its bandwidth, the more white noise it sees, and as a result it will have a higher noise floor. As an example, the noise floors for various bandwidth PSPL-LeCroy sampling heads range from 0.5 mV(rms) for a 20 GHz sampler to 3 mV(rms) for a 100 GHz sampler [2].

Signal averaging of measured sampled voltages can give considerable improvement in the noise floor of an equivalent time, sequential, sampling oscilloscope. Key to such oscilloscopes is the fundamental requirement that the waveform to be measured is not a single transient, but instead is a recurring, unchanging, waveform. Thus, if one is willing to use a longer data acquisition time, many reoccurrences of the same signal can be measured and averaged. The improvement in the noise floor due to signal averaging is given approximately by the square root of the number of samples averaged, Ns.

$$V_{nf}(avgd) = V_{nf} \; / \; (Ns)^{1/2}$$

There is, however, a practical upper limit to signal averaging. This is when other, non-random, long-term effects start to dominate the measurement. These can be such thermally driven effects as dc offset drifts and gain drifts.

JITTER: 'Jitter' is the commonly used term to describe the horizontal, or timing, uncertainty in exactly where in time a particular voltage sample is taken. Jitter occurs due to noises in the oscilloscope's various timing circuits, including the trigger, time base delay and sweep, and the sampling strobe generator. Jitter can also be present in the signal source of the unknown waveform to be measured. In this case, jitter could be present between the signal generator's trigger output and the main pulse output.

Jitter is particularly noticeable when observing very fast pulse edges. If the jitter's width is comparable to the pulse rise or falltime, then accurate waveform measurements become difficult to make. Figure 1 shows such a jittery waveform.

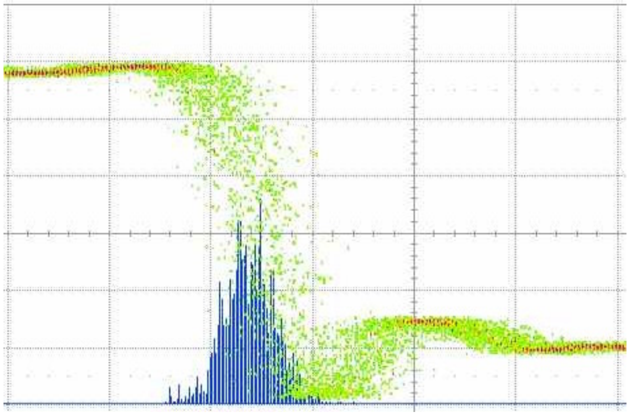

Fig. 1 A pulse edge with Jitter.
Blue trace is jitter histogram. 10 ps/div

Users of sampling oscilloscopes oftentimes will turn on the signal averaging feature in an attempt to improve their measurement of a jittery waveform. While the result may be pleasing to the eye, the accuracy is compromised.

Vertical signal averaging of a time jittered waveform results in distortion and smearing of the displayed waveform.

Gans and Andrews at NBS [3] first showed in 1975 that using signal averaging on a jittered waveform is equivalent to adding another low pass filter to the oscilloscope, thus lowering its effective bandwidth and slowing its risetime. Gans' 1986 Proceedings of IEEE paper [4] is the most commonly referenced paper on the bandwidth limiting effects of signal averaging jitter.

To demonstrate in a precisely controlled manner the effects of signal averaging on a jittery pulse waveform, the author has written several MatLab [5] programs to generate and analyze pulse waveforms with various controlled amounts of both vertical noise and horizontal, timing jitter. The most revealing results were obtained using computer generated Gaussian steps and impulses with added distortions of overshoot and ringing. For the purposes of this paper, the examples will be limited to a 100 ps step in a 1 ns time window. See Figures 2, 3, and 4.

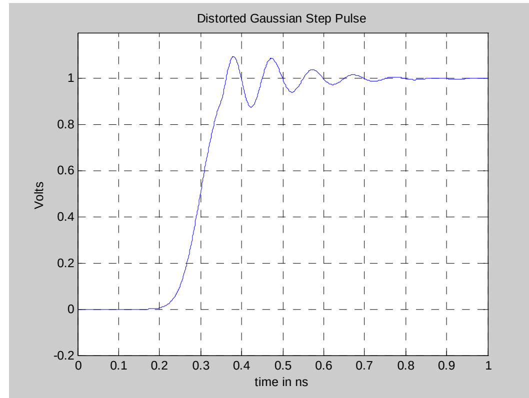

Fig. 2 Ideal test signal — a 1V, 100 ps rise Gaussian step with added overshoot and ringing

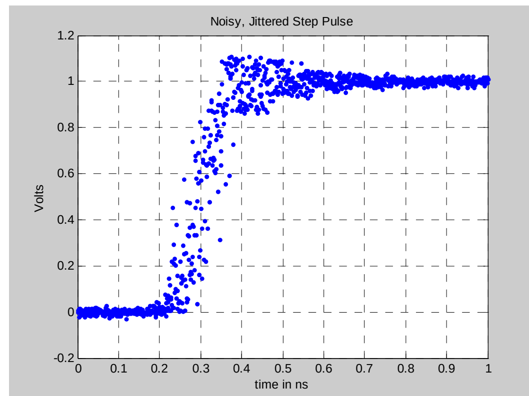

Fig 3 Distorted step pulse of Fig. 2, but with added vertical noise of 10 mV and 25 ps timing jitter

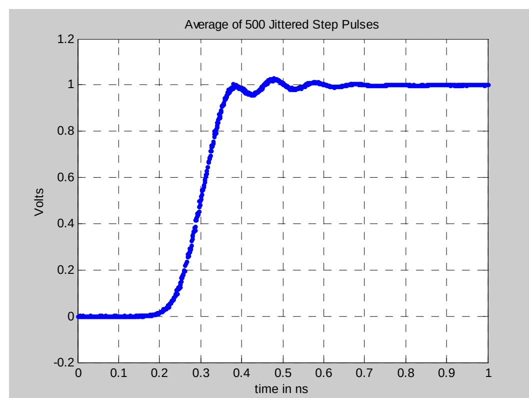

Fig. 4 The result of signal averaging 500 noisy, jittery waveforms.

Comparing Figure 2 and Figure 4, it should be obvious that, while signal averaging gave a nice, clean appearing step pulse waveform, it also slowed down the risetime from 100 ps to 111 ps, eliminated the overshoot, and severely damped the ringing.

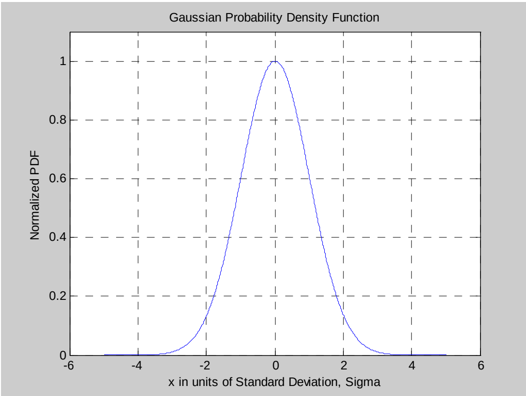

Fig. 5 Gaussian probability density function

STATISTICS OF NOISE: Many random noise processes are described by the 'Normal' or 'Gaussian' Probability Density Function (pdf) [6], Figure 5.

$$pdf(x) = [1/\sigma(2*\pi)^{1/2}]*\exp[-0.5*(x/\sigma)^2]$$

where σ is the standard deviation.
The Cumulative Distribution Function (cdf) is the integral from -∞ to x of the pdf, Figure 6.

$$cdf(x) = \int_{-\infty}^{x} pdf(x)\, dx$$

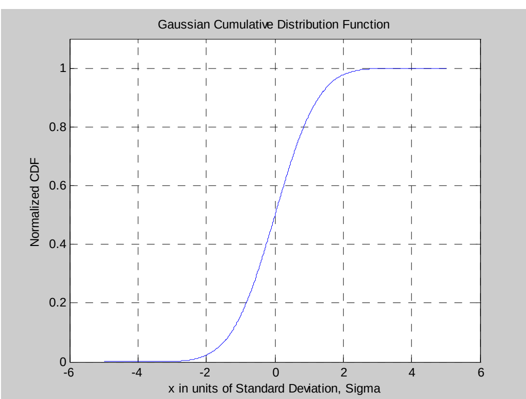

Fig. 6 Gaussian cumulative distribution function

Modern digital sampling oscilloscopes from LeCroy, Agilent, and Tektronix all include the capability of performing histogram analysis of both vertical (voltage) and horizontal (time) data. If the resulting histogram is similar in shape to the Gaussian pdf of Figure 5, then one can safely assume that the noise or jitter process is in fact Gaussian. Figure 1 shows a jitter histogram performed on a LeCroy oscilloscope.

Jitter experiments performed by the author at NBS [3] in the mid 70s used a large number of independent observers to give 'eye-ball' estimates of the 'width' of a jittery signal, such as shown in Figure 1. The results were that most human observers said the width of a jittering signal was 3 * σj, where σj was the actual standard deviation of the jitter. From Figure 6, we see from the cdf that these observations include about 87% of all the occurrences.

GAUSSIAN SIGNALS: Many of the pulse waveforms encountered in real life resemble the Gaussian impulse and Gaussian step shown above in Figures 5 and 6. In addition, oscilloscope manufacturers strive to make their instruments have Gaussian-like transient responses. The key parameter that completely describes these waveforms is σ, the standard deviation. It is thus worthwhile to here summarize the relationships between the various pulse parameters that are oftentimes measured.

Gaussian Step —
Trise(10-90%) = 2.564 * σ
Trise * BW(-3dB) = 0.338

Gaussian Impulse —
Tduration = fwhm(50%) = 2.354 * σ
Trise(10-90%) = Tfall = 1.688 * σ
BandWidth BW(-3dB) = 0.132*(1 / σ)

BACK-OF-THE-ENVELOPE CALCULATIONS: For many pulse measurements, the ultimate accuracy is not required. A very commonly made measurement is that of Risetime. Most engineers and technicians are familiar with the 'root-sum-of-squares' equation for risetimes.

$$T_r(out) = [\sum T_r(i)^2]^{1/2}$$

As an example, if one used a PSPL/LeCroy 70 GHz sampler with a 4.8 ps risetime to measure a pulse generator with a 7 ps risetime, the resultant displayed risetime would be 8.5 ps, or +21% in error.

Now assume that timing jitter was also present and signal averaging was used. Now the RSS equation must also include the effects of jitter.

$$T_r(out) = [T_r(gen)^2 + T_r(scope)^2 + T_r(jitter)^2]^{1/2}$$

From the above Gaussian relationships, we see that

$$Tr\ (jitter) = 2.564 * \sigma j$$

where σj is the timing jitter standard deviation.

For the above example, if the timing jitter is 1.5 ps, then the effective averaged jitter risetime is 3.9 ps, and the effective bandwidth of the averaged jitter low pass filter is 88 GHz. The risetime of the averaged, jittery waveform would be 9.3 ps, or +33% in error.

Thus, when timing jitter is present and signal averaging is used, this RSS equation should be used.

$$T_r(gen) = [T_r(meas)^2 - T_r(scope)^2 - (2.56 * \sigma j)^2]^{1/2}$$

When dealing with impulsive like signals, the key parameter is the duration, T_d, or sometimes referred to as the 'full-width-half-max' (fwhm). To calculate durations, the RSS equation can still be used, but the various risetimes should not be used. Instead, the impulse response durations should be used. For Gaussian networks, the impulse duration and step risetime are related by

$$T_d = fwhm = 0.918 * T_{rise}$$

$$T_d(gen) = [T_d(meas)^2 - T_d(scope)^2 - (2.35 * \sigma j)^2]^{1/2}$$

Whenever impulses are passed through low pass filter networks, such as an oscilloscope's response, or signal averaged jitter, the impulse will always lose its peak amplitude, V_{pk}, and it will also be smeared out with a longer duration (fwhm). However, the area of the impulse will remain relatively constant. The area can be approximated by:

$$Area = V_{pk} * T_d$$

Thus, once one has determined the input duration, the input peak amplitude can also be calculated.

$$V_{pk}(gen) = [T_d(meas) / T_d(gen)] * V_{pk}(meas)$$

DECONVOLUTION OF JITTER: A more accurate way to remove the effects of signal averaged timing jitter is to use deconvolution. We have seen that signal averaged jitter is equivalent to inserting a Gaussian low pass filter into the measurement chain. Because it is Gaussian, we know the frequency response, transfer function, for this filter to be

$$H_j(f) = \exp[-0.5 * (f / \sigma_f)^2]$$

where $\sigma_f = 1 / (2 * \pi * \sigma_t)$

σ_t is the jitter standard deviation in seconds, while σ_f is the equivalent frequency standard deviation in Hertz.

Let $v_{out}(t)$ be the averaged jitter waveform and $v_{in}(t)$ be the true waveform without jitter. Then deconvolution yields

$$Vin(f) = Vout(f) / Hj(f)$$

where $Vout(f) = FFT[v_{out}(t)]$

and $v_{in}(t) = invFFT[Vin(f)]$

A MatLab computer program, *JitterDeconV1.m*, has been written to perform deconvolution of jitter. This program is available from PSPL.

Using this program on the signal averaged step waveform in Figure 4 gave the results shown in Figure 7. The original waveform, Figure 2, had 100 ps risetime and 10% overshoot. Vertical noise and a lot of timing jitter were then added, with the result shown in Figure 3. The jitter standard deviation, σ, was 25 ps, and thus the visual width (3*σ) of the jitter was of the same order as the risetime of the original signal. Looking at the jittered waveform, it is impossible to discern hardly any of its features. Signal averaging improved the resolution but also slowed the risetime from 100 ps to 111 ps. The deconvolved waveform (red trace, Figure 7) has restored many of the characteristics of the original waveform, with very good agreement on the leading edge. The reconstruction is not perfect, but is much better than the signal averaged trace. When the jitter is much less than the fastest transient in the signal, the reconstruction is almost ideal.

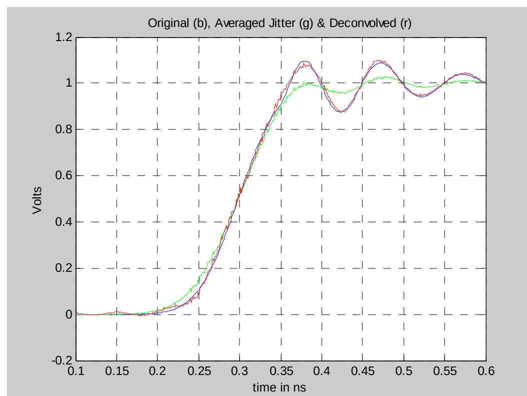

Fig. 7 Jitter Deconvolution —
Blue trace is true, original, jitter-free waveform.
Green trace is the signal averaged, jittery waveform.
Red trace is the deconvolved estimate of the original waveform. σn = 10 mV and σj = 25 ps

Experiments were run using various amounts of vertical noise and horizontal timing jitter. The conclusions were:

1. If the visual jitter (3*σ) is > 150% * Trise, then jitter deconvolution will not work.
2. If the visual jitter (3*σ) is < 20% * Trise, then jitter deconvolution is not necessary.
3. Large amounts of vertical noise up to a s/n of 6 dB had minimal effect on the jitter deconvolution process.

Care must be taken in the deconvolution process to not perform the division by Hj(f) in those regions where Vout(f) is in the noise floor. Doing so will cause the noise floor to rise dramatically and the vin(t) solution to explode. Figure 8 shows the frequency spectrums for Vout and Vin. The spectrum of Vout reached the noise floor of -15 dB at a frequency of 18 GHz. The program *JitterDeconV1.m* determines the noise floor and will not apply H(f) correction there.

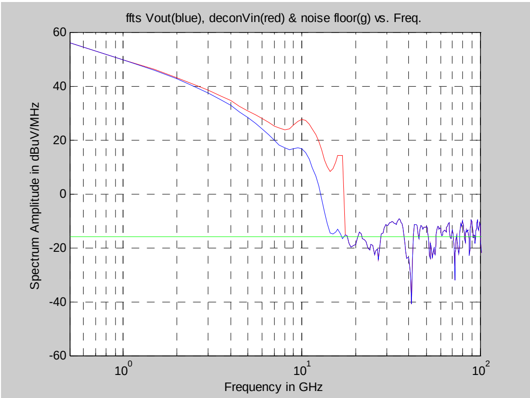

Fig. 8 Frequency spectrum of Vout(f) (blue) and deconvolved Vin(f) (red)

To learn more about deconvolution, the reader is referred to PSPL's application note AN-18 [1], which is available from the PSPL web site www.picosecond.com.

REFERENCES

[1] J.R. Andrews, "Deconvolution of System Impulse Responses and Time Domain Waveforms", Application Note AN-18, Picosecond Pulse Labs, Boulder, CO, Oct. 2004

[2] LeCroy web site: www.lecroy.com WaveExpert model 100H oscilloscope

Note: PSPL designed and builds the LeCroy SE, ST and SO series sampling heads on an OEM basis.

[3] W.L. Gans and J.R. Andrews, "Time Domain Automatic Network Analyzer for Measurement of RF and Microwave Components", NBS Tech Note 672, National Bureau of Standards (now NIST), Boulder, CO, Sept. 1975. See section 9.3 Random Errors, pp. 109-124.

[4] W.L. Gans, "Calibration and Error Analysis of a Picosecond Pulse Waveform Measurement System at NBS", Proc. of IEEE, vol. 74, no. 1, pp. 86-90, Jan. 1986

[5] MATLAB, Math Works, Natick, MA, www.mathworks.com

[6] Reference Data for Engineers, 7th edition, 1985, H.W. Sams & Co., chp. 44 "Probability & Statistics", pp. 44-4 & 44-5