

## Time Domain Spectrum Analyzer and "S" Parameter Vector Network Analyzer

James R. Andrews, Ph.D., IEEE Fellow  
PSPL Founder & former President (retired)

**INTRODUCTION** In the frequency domain, two very commonly used electrical test instruments are the Spectrum Analyzer (SA) and the Vector Network Analyzer (VNA). The spectrum analyzer is an electronically tuned radio receiver that gives an X-Y plot of signal strength vs. frequency. An elementary scalar network analyzer can be built with a spectrum analyzer in conjunction with a sine wave signal generator that is slaved to the analyzer to track it in frequency. If the radio receiver(s) in the network analyzer includes both amplitude and phase detectors, then the network analyzer is capable of complex measurements of both amplitude and phase and is then called a VNA. Spectrum analyzers and VNAs are typically very expensive instruments. In the time domain, the key measurement instrument is the oscilloscope, which gives an X-Y plot of voltage vs. time. Typically, oscilloscopes are less expensive instruments than spectrum analyzers or VNAs. Pairing up an oscilloscope with a low cost, digital computer results in a very powerful instrument that can also be used as a spectrum analyzer and VNA [1-4]. This application note will present the necessary math concepts, including the FFT, that are used to transform time domain data from the oscilloscope into frequency domain spectrum and vector network measurements. Time domain spectrum analyzer and VNA examples plus MATLAB programs are also included.

**TD / FD MATH** A key mathematical concept that is useful for understanding many real-world engineering systems are the transformations back and forth between the Time Domain (TD) and the Frequency Domain (FD). The 18th century French mathematicians Fourier and Laplace were the first to observe and formalize into mathematics these relationships. All engineering students have studied this subject in detail. We will not review all of this material except to recall a few of the key concepts. A good web resource for a quick refresher is found in reference [5].

The Fourier Series deals with periodic signals. These are time domain signals that start at  $t = -\infty$  and go to

$t = +\infty$ . A perfect replica of the signal repeats itself infinitely with a period of  $T_p$ . Fourier showed that a perfect replica of this periodic signal can be synthesized with an infinite series of sine and cosine waves. This is called the Fourier Series. The lowest frequency in the series (besides DC) is  $f_0$ , which is the reciprocal of the period,  $T_p$ , i.e.,  $f_0 = 1/T_p$ . All other frequencies are integer harmonics of  $f_0$ , i.e.,  $f_i = i * f_0$ . The recreation of the time domain signal is controlled by the amplitude and phase of each harmonic frequency. The result in the frequency domain is a non-continuous, discrete signal that only exists at specific, harmonically related frequencies. When observed on a spectrum analyzer, these appear as a 'comb' of 'spikes'.

The Laplace transform deals with single transient signals. These are signals that do not exist for  $t < 0$ . At  $t = 0$  or some later time, they start and continue on to  $t = +\infty$ . The Laplace transform converts the time domain to the complex frequency domain, where the real axis is the commonly understood frequency ' $\omega$ ', where  $\omega = 2 * \pi * f$ . The result in the frequency domain is a continuous function, not a discrete set of 'spikes' as found with a periodic signal.

The Fourier Transform is a cross between the Fourier Series and the Laplace Transform. If the time domain signal is no longer a periodic signal in the conventional sense, extending from  $t = -T_p/2$  to  $t = +T_p/2$ , then for the Fourier Transform, these limits are extended out to both  $-\infty$  and  $+\infty$ . Again, the result in the frequency domain becomes a continuous function, not a discrete set of 'spikes' as found with a periodic signal.

**DFT / FFT** We never have the luxury of being able to make measurements from  $t = -\infty$  to  $+\infty$  with infinite resolution. Instead we are constrained to making a finite set of measurements. Typically, we will take a set of  $N$  sampled measurements over a window of time,  $T_w$ , with a time spacing of  $\Delta t$  between each measurement.

$$T_w = N * \Delta t \quad (1) \quad \text{or} \quad \Delta t = T_w / N \quad (2)$$

From this set of N measurements, we can still obtain frequency domain information about the signal under test. The process for doing this is called the Discrete Fourier Transform (DFT). There are a few key limitations that must be understood about the DFT. The first inherent assumption in the DFT is that it actually is a discrete Fourier Series, not a transform from  $t = -\infty$  to  $+\infty$ . The DFT assumes that the signal being sampled is actually a periodic signal of period  $T_p = T_w$ . Thus, from this we see that the fundamental frequency resolution,  $\Delta f$ , of the DFT will be limited to the fundamental frequency.

$$\Delta f = f_0 = 1/T_w \quad (3)$$

The next limitation is that the high frequency coverage does not go out all the way to  $\infty$ . Because our minimum spacing between sampled measurements is  $\Delta t$ , we will not be able to measure any frequencies higher than the Nyquist frequency,  $f_{ny}$  where:

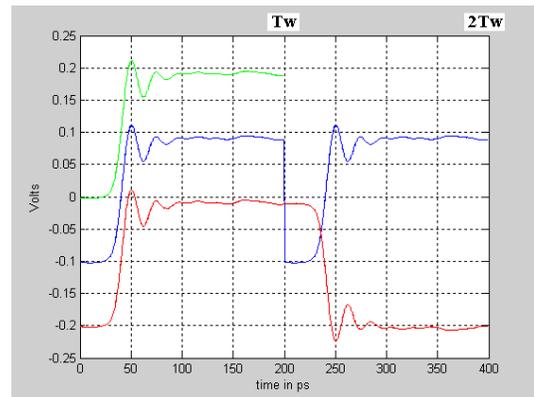
$$f_{ny} = 1 / (2 * \Delta t) \quad (4)$$

Graphically this can be visualized as saying that for a sine wave of frequency  $f_{ny}$  there are only three data points on one period, and they all hit the same level. For any sine wave of  $f > f_{ny}$ , there will be fewer than three data points, and they will be insufficient to describe the sine wave. When a DFT is performed on a set of N time domain data points, a frequency domain array of size N results. The first half of the array contains the dc term and the harmonically related positive frequencies from  $f_0$  up to  $f_{ny}$ . The second half of the array contains redundant information from the 'negative' frequencies from  $-f_{ny}$  back to  $-f_0$ . The 'negative' frequencies are the complex conjugate of the 'positive' frequencies.

The Fast Fourier Transform (FFT) is a sub-set of the DFT. It is a fast computer algorithm developed by Cooley & Tukey in 1965 [6] that reduces the number of computations needed for N points from  $2N^2$  to  $2N * \log_2(N)$ . If N is a power of 2 (i.e., ... 32, 64, 128, ... etc.), then the speed of computation is optimized.

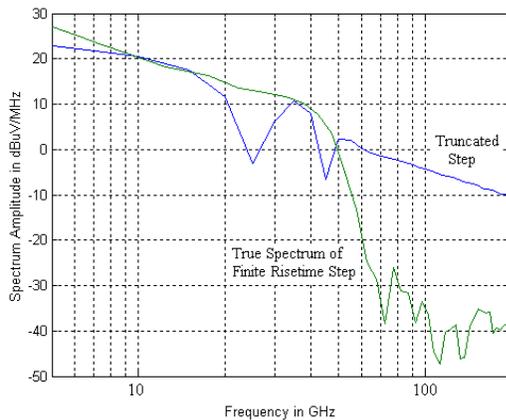
**WINDOWING** There is a major problem in converting time domain data to the frequency domain using the DFT. What is the effect of the signal that lies outside of the time window of observation? In the ideal situation, the time window is large enough to contain all of the signal, in which case both the signal and its derivatives are zero outside of  $T_w$ . When this is not the case, then truncation of the signal outside of the time window contributes errors into the computed DFT. Depending upon the nature of the signal and the

truncation, these errors can be small or very significant. How should one minimize its effect and maximize the accuracy in the measurements? There has been a great deal of research with lots of publications on this subject. This is oftentimes dealt with by multiplying the measured data set by a Window function (also called a tapering or apodization function [5]). The window function is a symmetrical function whose purpose is to force the absolute value and oftentimes the first derivative,  $dv/dt$ , to zero at the beginning and end of the data set. The choice of a particular window function depends upon the particular equivalent frequency domain instrument function to be optimized, such as frequency resolution, amplitude accuracy, filter side lobe levels, etc. Among some of the popular window functions are the Uniform (or rectangular window), Cosine, and Hanning. The Cosine applies a single half cycle of a cosine wave to the measured data set, thus forcing the end points to zero. The Uniform is really the 'do nothing' function. It is used when dealing with a nearly ideal impulse-type signal in which the beginning and end points of the measured data set are already almost identical.



**Fig. 1** Various step signals for DFT/FFT processing. The example shown is a PSPL model 4022, 7.5 ps, TDR pulser measured by an HP 50 GHz oscilloscope.

Probably the worst possible case signal to deal with for a DFT is the Unit Step function. See Figure 1. The top (green) trace is a typical data acquisition of a finite risetime, step-like waveform. When this data is processed by the DFT or FFT, it is converted into a periodic signal as shown by the middle (blue) trace. This introduces a very large truncation error at the end of the data set, with a very large discontinuity resetting from the topline back to the baseline in one sampling interval,  $\Delta t$ . This effectively introduces an artificial falltime of  $\Delta t$  into the waveform processed by the DFT/FFT.



**Fig. 2** Spectrums of Step-Like Waveforms. The example shown is a PSPL model 4022 TDR pulser measured by an HP 50 GHz oscilloscope.

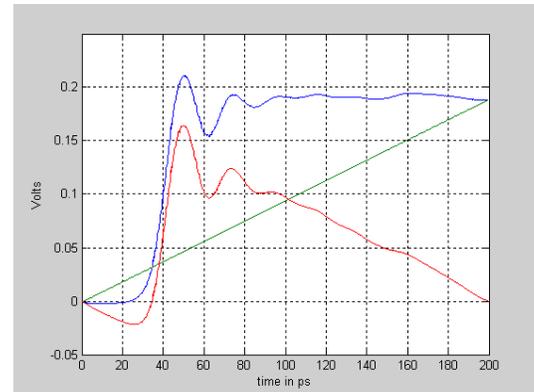
Figure 2 shows the spectrums for the two different 'step' waveforms (top and middle) from Figure 1. The green trace is the true spectrum of a finite risetime step, while the blue trace is that of the truncated step. It is obvious that the large discontinuity truncation at the end of the data set introduced huge errors at high frequencies. The high frequencies required in a Fourier Series to reproduce the  $\Delta t$  falltime discontinuity are totally false information.

In the 1970s, in the early days of digital signal processing, the author introduced a concept different from a window function for dealing with the truncation error of a step function type signal [4]. The author's concept was to first convert the step function signal into a perfectly symmetrical square wave-type signal prior to DFT processing. See the bottom (red) trace in Figure 1. The data set of  $N$  points is inverted, time shifted by  $T_w$  and added to the end of the original data set, resulting in a new data set of  $2*N$  and a time window of  $2*T_w$ . The sample spacing,  $\Delta t$ , and the Nyquist frequency remain the same. The argument for the validity of this operation is: If one is dealing with a linear system, it will respond identically, except for polarity, to a step function excitation that is of either positive or negative polarity. The result in the frequency domain becomes a spectrum display that starts now from a lower frequency of  $1/(2T_w)$  and only contains the odd harmonics of  $1/(2T_w)$ .

Unfortunately this square wave idea was not picked up and used much by other researchers, the reason being that it doubled the size of the data array for computer processing. In the 1970s, this was a big deal because computers of that era had very limited memory capacity. Today, with modern PCs, memory size and processing speed is no longer an issue. The MATLAB programs included in this application note use this

technique for dealing with step function type signals. Gans, a co-author of [2], published a short paper [7] on the 'square-wave' method in 1982, which received wider distribution. As a result this method has since become known in the scientific literature as the 'Gans Square-Wave' method.

Another technique to remove the step function  $\Delta t$  falltime discontinuity that has been widely used is the 'Nicolson Ramp Subtraction' [8]. This involves subtracting from the step-like waveform a linear ramp whose beginning and end points match the step waveform. This has the effect of forcing the beginning and end points down to zero. Figure 3 shows the waveforms. DC information is lost. Waldmeyer [9] has theoretically shown that the spectrums resulting from the Gans and Nicolson methods are the same. Riad [10] has gone one step further. He proposed to merge into one composite spectrum the results from both the Gans and Nicolson methods. Because the Gans square-wave method results in nulls at the even harmonics, Riad then inserted at the null frequencies the spectrum computed by the Nicolson ramp subtraction method. The Riad composite technique [10] for treating step-like waveforms is what is used in the PSPL spectrum and network analysis programs.



**Fig. 3** The Nicolson method [8] of subtracting a ramp (green) from a step-like waveform (blue) to create a windowed waveform (red) suitable for FFTing.

**$N$ ,  $T_w$ ,  $f_o$ ,  $\Delta t$ , and  $f_{ny}$**  When setting up the time domain measurements used to calculate the frequency spectrum or VNA parameters, the experimenter will have the choice of the oscilloscope's time window of observation,  $T_w$ , and the number of data points,  $N$ . From  $N$  and  $T_w$  then,  $f_o$ ,  $\Delta t$ , and  $f_{ny}$  result from equations (2-4). Oftentimes compromises must be made either due to instrument limitations and/or the spectral coverage of the signal being too broad. The author has found that a good rule of thumb for

adequate frequency coverage is to select the sample spacing,  $\Delta t$ , to include at least five or more samples on the fastest transition present on the waveform. If that is the step risetime, then select

$$\Delta t < T_{\text{rise}} / 5 \quad (5)$$

To enhance the frequency resolution,  $f_o$ , the time window,  $T_w$ , should be set as wide as possible along with the largest possible data set,  $N$ , consistent with instrument and computer limitations.

**SPECTRUM ANALYZER PROGRAM** A program, called *SpecAnalysis.m*, for spectrum analysis of time domain waveforms is available upon request. This program was written in MATLAB [11] by the author. To make this program independent of the data format acquired from any particular digital oscilloscope, the data entry into the program is via a \*.txt file from either a floppy disc in A: drive or the current working directory on the C: drive. Also, the program queries the operator for some supplemental data such as the Time Window,  $T_w$ , for entry via the keyboard. This program uses the uniform, rectangular (i.e., 'do nothing') window function as its default window.

The program queries the operator for the signal type, either periodic, impulse, or step. The first choice, 'periodic', is for any arbitrary, periodic waveform. The only limitation is the requirement that the time window of data acquisition must include at least one cycle of the waveform. The second choice, 'impulse', is used with single transient, Laplace-like waveforms such as impulses. The only limitations are the requirements that the time window must be large enough to include the entire transient waveform and that the waveform has returned back to its baseline. The third choice, 'step', is intended for use with single transient, step-like waveforms. The only limitations are the requirements that the time window must be large enough to allow the transients in the step-like waveform to die out and that the step pulse has almost reached its final value.

The *SpecAnalysis.m* program plots the input waveform and the calculated spectrum. The calculated results can optionally be stored to disc.

It should be noted that the calculated FFT results are plotted in different units, depending upon the signal type selected. For a 'periodic' signal, the units are the normally expected units of either Volts or power (dBm). These are the units associated with a periodic signal lasting from  $-\infty$  to  $+\infty$ . For a single transient, Laplace-like signal such as a single 'impulse' or 'step'

function, the appropriate units are not Volts, but Spectral Amplitude in Volt-Seconds or a spectral density of Volts per Hertz or, as used in these programs, dB $\mu$ V/MHz. For a detailed explanation of these differences, see the NBS reference [2].

For examples showing the use of this program, the reader is referred to PSPL's application note, AN-9 [12]. This includes a more detailed discussion of the theoretical spectrums of various pulse waveforms. Time domain plots are included for several ultra-fast, picosecond risetime, PSPL pulse generators and the computed FFT spectrums showing frequency content well past 50 GHz. They include step, impulse, and monocycle waveforms.

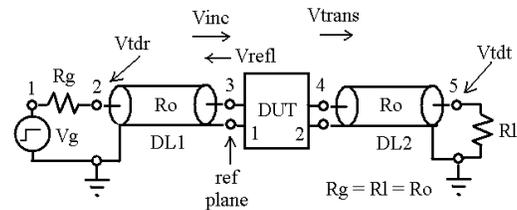


Fig. 4 Basic TDVNA Measurement Set-up

**VECTOR NETWORK ANALYZER** Figure 4 shows the basic test set for a Time Domain Vector Network Analyzer (TDVNA) [3,4]. It is also the basic test set for making TDT and TDR measurements. PSPL's TDR/TDT application note, AN-15 [13], should also be referred to at this time as it compliments this application note. Figure 4 is also the classical setup used in the frequency domain for making S parameter insertion and reflection measurements of  $S_{21}$  and  $S_{11}$ . A sine wave generator is used as the signal source ( $V_g, R_g$ ) for frequency domain measurements. For time domain measurements, a pulse generator is used as the signal source, and an oscilloscope is used to observe and measure the resultant waveforms,  $V_{tdr}(t)$  &  $V_{tdt}(t)$ . Measurements are usually standardized to the characteristic impedance,  $R_o$ , of the interconnecting coaxial cables and connectors with  $R_g = R_l = R_o$  (typically 50  $\Omega$ ). The measurement reference plane is at the far, right end of the signal source cable, DL<sub>1</sub>, node 3. To measure other S parameters such as  $S_{12}$  and  $S_{22}$ , simply reorient the DUT in the test set. For DUTs with more than two ports, all of the other ports must be terminated in  $R_o$ .

A set of Time Domain Transmission (TDT) measurements are made of  $V_{trans}$  to determine the 'S' parameter,  $S_{21}$ . For TDT measurements, the oscilloscope measures the waveforms at node 5. The system is first calibrated without the Device Under

Test (DUT) and with the output cable, DL<sub>2</sub>, (node 4) connected directly to the input cable, DL<sub>1</sub>, (node 3). The transmitted input test pulse waveform to node 5, V<sub>in</sub>, is observed and stored in memory. Next the DUT is inserted between nodes 3 and 4, and a new transmitted waveform to node 5, V<sub>out</sub>, is observed and stored in memory. The DUT is characterized by its transfer function, h(t). The output waveform, V<sub>out</sub>(t), is the convolution of the input waveform, V<sub>in</sub>(t), with h(t).

$$V_{out}(t) = V_{in}(t) ** h(t) \quad (6)$$

where \*\* denotes the convolution operator. In the frequency domain, the transfer function, H(f), is the forward scattering parameter, S<sub>21</sub>(f).

$$H(f) = S_{21}(f) = V_{out}(f) / V_{in}(f) \\ = \text{FFT}[V_{out}(t)] / \text{FFT}[V_{in}(t)] \quad (7)$$

To determine the reverse scattering parameter, S<sub>11</sub>, a set of Time Domain Reflectometry (TDR) measurements are made of waveforms V<sub>inc</sub> and V<sub>refl</sub>. The reflection coefficient ρ is defined as:

$$\rho = V_{refl}(t) / V_{inc}(t) \quad (8)$$

For TDR measurements, the oscilloscope measures the waveforms at node 2. The system is first calibrated without the DUT connected, and a short circuit is attached to node 3. The reflected, input test pulse from the short circuit, V<sub>short</sub>, is observed at node 2 and stored in memory. Using a short circuit allows us to measure a mirror image of V<sub>inc</sub>. Recall that the reflection coefficient of a good short circuit is exactly -1. An open circuit termination is not typically used because its reflection coefficient is not exactly +1 due to the fact that some energy is radiated from an open and is not perfectly reflected. Next the DUT is inserted and attached to nodes 3 and 4, and a new reflected waveform to node 2, V<sub>tdr</sub>, is observed and stored in memory. From these two waveforms, we can now compute in the frequency domain the reverse scattering parameter, S<sub>11</sub>.

$$S_{11}(f) = \text{FFT}[V_{tdr}(t)] / \text{FFT}[-1 * V_{short}(t)] \quad (9)$$

To make accurate TDVNA measurements, there are several practical considerations that should be adhered to. These are similar in concept to the elaborate calibration procedures that are done with frequency domain VNAs. The issues relating to the choices of T<sub>w</sub>, Δt, and N were discussed earlier.

Errors are introduced by system noise and drifts. Timing jitter is equivalent to phase noise errors and leads to a loss of high frequency information. Jitter

needs to be minimized as much as possible through selecting a low jitter pulse generator and oscilloscope and optimizing the trigger settings for minimum jitter. The system amplitude noise is usually set by the front end noise of the oscilloscope. After transformation via the FFT, this in turn sets the noise floor in the frequency domain. Most digital oscilloscopes include the capability of signal averaging. This should definitely be used to enhance the measurement's signal to noise ratio (S/N). There will be a practical upper limit in the amount of signal averaging possible. Eventually, if the measurement acquisition time becomes too long, then long term vertical and timing drifts will start to corrupt the data. These are usually due to thermal influences. Vertical drifts can be minimized by a fresh vertical calibration of the oscilloscope immediately prior to a critical measurement. Timing drifts are especially deadly as they destroy high frequency information in the data.

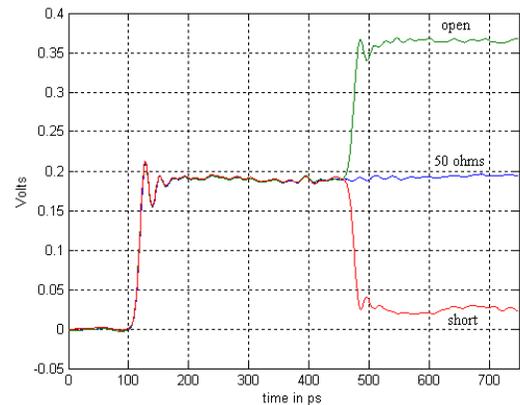
We have assumed that all impedances in the test set, Figure 4, are well matched to R<sub>o</sub>, thus eliminating multiple reflections between the source, DUT, load, etc. For low frequency instruments, they are usually well matched to 50 Ω. For high frequency, fast risetime (< 1 ns) instruments that are claimed to nominally have a 50 Ω impedance, this is not always a good approximation. Most digital sampling oscilloscope manufacturers strive to match the DC input resistance of their instrument to within ± 2 % of 50 Ω. Even with the DC being matched, there is still typically a small shunt capacitance due to the sampling bridge that leads to mismatch at high frequencies. However, we have observed that the DC input resistance of one manufacturer's 70 GHz oscilloscope was actually 70 Ω (i.e., a severe mismatch ρ of 29%). Pulse generator sources are usually not as well matched to 50 Ω as are oscilloscopes. In the frequency domain, for single frequency measurements, one usually uses tuning networks on the source and load to eliminate multiple reflections. For broadband VNAs, corrections for source and load mismatch are accomplished in software via elaborate calibration routines. For the TDVNA, the preferred approach to matching the source and load to the DUT is accomplished using Time Window Isolation Lines. This is the key purpose of the two transmission lines, DL<sub>1</sub> and DL<sub>2</sub>, shown in Figure 4. Very high quality coax cables should be used for DL<sub>1</sub> and DL<sub>2</sub>. They should be very low loss to minimize the smearing of pulse edges. They should also be extremely closely matched to 50 Ω. The length of DL<sub>1</sub> and DL<sub>2</sub> should be chosen such that the two way transit time through each line is just slightly longer than the measurement time window, T<sub>w</sub>. Obviously one also needs to use high quality 50 Ω coax connectors. Without DL<sub>1</sub> and DL<sub>2</sub>, measured data will

be compromised due to multiple reflections between the DUT,  $R_g$  and  $R_l$ . Another impedance matching technique sometimes used is to pad the source and oscilloscope with high quality, low vswr, coaxial attenuators. This is only useful if one has a high output amplitude pulse generator that needs to be attenuated anyway.

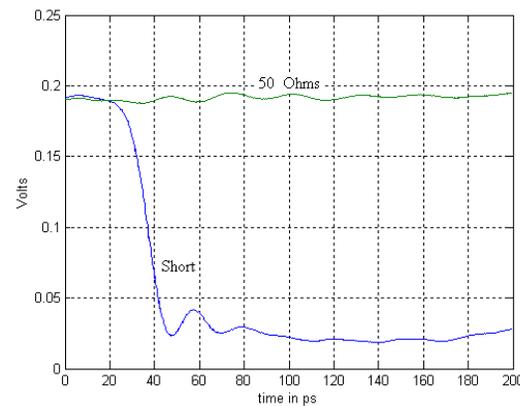
An often overlooked source of error is baseline correction. The author has found that every sampling oscilloscope tested had a low level non-zero baseline signature waveform. This is found by connecting a 50 $\Omega$  termination instead of a signal source to the input of the oscilloscope. Next take a lot of signal averages, and then magnify vertically the trace. Ideally it should be a perfectly flat, 0 V baseline with only random noise. Instead, one will usually discover a low level, fixed pattern waveform along with the random noise. This pattern is due to crosstalk from other electrical signals within the oscilloscope. The pattern will be dependent upon the time window and also the position of the time delay (position) controls. To enhance  $S_{21}$  measurements, the non-zero baselines need to be subtracted from the measured data.

In  $S_{11}$  TDR measurements, there is also a baseline problem, but it is usually dominated by irregularities in the pulse generator waveform rather than the oscilloscope. This baseline is determined by measuring the waveform when the TDR is terminated in 50 $\Omega$ . For  $S_{11}$  TDR, one needs to use two standards to calibrate the TDVNA. See Figure 5. The first standard mentioned earlier was the short circuit. The second standard is a known, good, low vswr ( $\rho=0$ ), 50 $\Omega$  termination. An excellent termination is oftentimes the second coax cable, DL<sub>2</sub>. The waveform from the 50 $\Omega$  termination should be measured, stored in memory, and subtracted from all measured TDR waveforms.

Another overlooked error is the proper positioning of the measured waveforms in the time window. See Figures 6 and 7. A portion of the zero signal baseline needs to be included; however, it should only occupy a small portion of the data window. The author usually positions his signals so that the impulse or step risetime occurs about 10% to 20% in from the left edge of the oscilloscope screen.



**Fig. 5** Calibration Standards for  $S_{11}$  TDR measurements. Short Circuit (red), Open Circuit (green), and 50  $\Omega$  Termination (blue)



**Fig. 6** Optimum positioning of TDVNA TDR,  $S_{11}$ , waveforms on oscilloscope screen

Figure 6 shows where TDR,  $S_{11}$ , waveforms should be positioned on the screen. The leading edge of the TDR step pulse is not measured and should be positioned off the screen to the left. A portion of the 50  $\Omega$  reference level should occupy the first 10%. The reflected edge from the short circuit calibration standard is placed 10-20% in from the left side.

When performing insertion  $S_{21}$  measurements, Figure 7, oftentimes the DUT has a large inherent physical time delay that could push its response waveform (green trace) to the far right side of the oscilloscope screen or even off it completely. A coax cable is a good example of such a network that might have a very fast risetime but a very long insertion time delay. The oscilloscope's horizontal delay (position) control should be adjusted to reposition the DUT's response waveform back towards the left side of the screen, (red trace). The optimum setting is to have the response pulse just start at the same position as the input excitation pulse. The basic delay of the DUT is thus

the difference in the two oscilloscope delay settings. This should be added onto any additional group delay measured by the TDVNA. When time shifting is used, a second time shifted baseline should be measured.

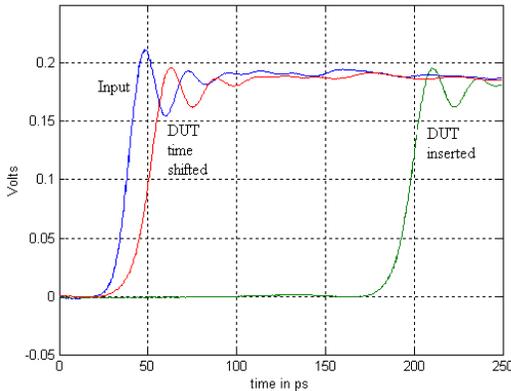


Fig. 7 Optimum positioning of TDVNA TDT,  $S_{21}$ , waveforms on oscilloscope screen

**TDVNA PROGRAM** A program called *TDVNA.m* for time domain vector network analysis is available upon request. This program was written in MATLAB [11] by the author. As with the spectrum analysis program, data entry is via \*.txt files from either the floppy disc A: drive or the current working directory on the C: drive and the keyboard. The test signal from the pulse generator must either be a impulsive type waveform or a step-like waveform. No window function is used; thus, the time window,  $T_w$ , must be long enough to allow the impulse to settle back to its baseline or the step to stabilize at its final value. If a step is used, the program automatically converts the data into both square waves and subtracted ramps prior to performing the FFTs. Both programs provide plots of the measured time domain data along with two calculated, frequency domain, output plots. The first is the magnitude of either  $S_{11}$  or  $S_{21}$  in dB vs. frequency. The second is the phase plotted as Group Delay vs. frequency. The calculated results can optionally be stored to disc.

**TDVNA EXAMPLES** The remaining figures in this application note will show some TDVNA examples of what can be accomplished using a PSPL model 4022 TDR/TDT pulse generator, producing a 200 mV, 7.5 ps risetime, step pulse, along with a Hewlett-Packard, 50 GHz, 8.8 ps risetime, digital sampling oscilloscope, plus the *TDVNA.m*, MATLAB program. The TDR pulse head was used as the signal source for both  $S_{11}$  and  $S_{21}$  measurements. The measured pulser/scope step risetime was 12.2 ps; see Figure 1. The measurement

system was configured using "K" (2.95mm) connectors. The time window isolation lines,  $DL_1$  and  $DL_2$ , were a pair of 35 cm Gore, test set, coax cables. Anritsu models 22K20B, 22K22B, and 22K24B "K" equi-phase, gender-changing, adapters were used on these cables at the test ports (nodes 3 and 4). The 50  $\Omega$  calibration standard was the second time window line,  $DL_2$ . The oscilloscope settings were as follows:  $T_w = 2$  ns (i.e., 200 ps/div),  $N = 4096$ ,  $\Delta t = 0.488$  ps, and 256 digital averages. The calibration edges were positioned 200 ps (10%) in from the left side. The overall measured system risetime for TDR/ $S_{11}$  was 17 ps and for TDT/ $S_{21}$  was 16.3 ps. The loss of  $\approx 4 - 5$  ps in risetime and the resultant bandwidth was due to the high frequency loss in the time window isolation lines,  $DL_1$  and  $DL_2$ . TDR reflected signals also suffered an additional slowing due to transiting through the feed-thru pulse head. This particular combination of instruments, components, and oscilloscope settings resulted in a 50  $\Omega$ , TDVNA with frequency coverage from 250 MHz to 45 GHz for  $S_{11}$  and 50 GHz for  $S_{21}$ .

If the time window isolation lines are not used, then this particular TDVNA system's bandwidth is enhanced. For TDT the risetime is 12.2 ps. For TDR the risetime is 15.0 ps. Frequency coverage is now up to 50 GHz for  $S_{11}$  and beyond 60 GHz for  $S_{21}$ .

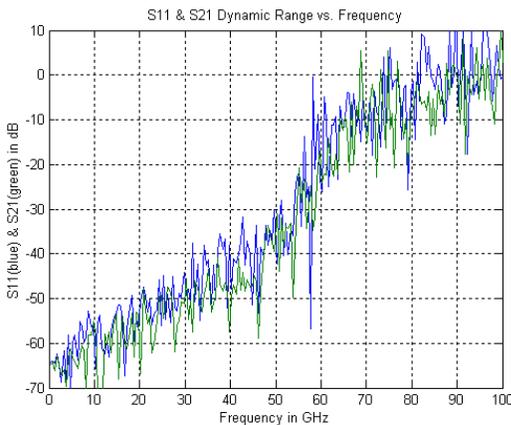
The reader is also referred to a recent paper by K. Schoen [14] that demonstrated an even higher bandwidth, 50  $\Omega$ , TDVNA useful to beyond 65 GHz. He used a PSPL model 4022 9 ps TDR and 5 ps TDT pulse generator along with an Agilent 70 GHz sampling oscilloscope. He opted for the maximum bandwidth and thus did not use time window isolation lines. His results were compared with measurements made on an Anritsu 65 GHz VNA. Excellent agreement was obtained to 30 GHz and acceptable agreement up to 65 GHz.

The author has also built a 75  $\Omega$ , 26 GHz, TDVNA using these same techniques. For details, see the PSPL application note, AN-17 [15].

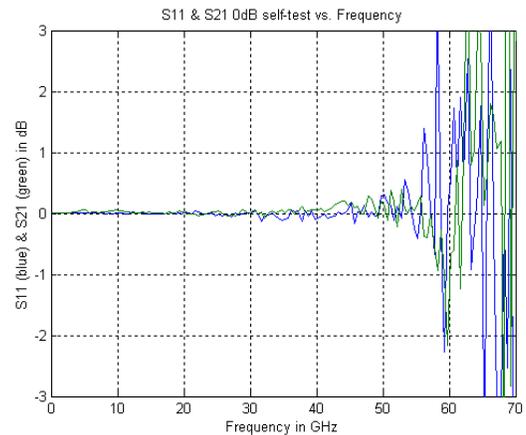
**TDVNA SYSTEM SELF-TESTS** When assembling a TDVNA, one should first perform a set of system self-tests to determine the system noise floor, dynamic range, repeatability, and useful frequency coverage. The first 'self-test' is dynamic range. For  $S_{21}$ , make a normal measurement of  $V_{out}$  but without the DUT installed. In its place connect a 50  $\Omega$  termination to node 4, Figure 3, and again measure the system baseline. For  $S_{11}$ , make another measurement of  $V_{tdr}$  with the 50  $\Omega$  standard termination still connected to node 3, Figure 3. Figure 8 shows the dynamic range of

the subject TDVNA. At very low frequencies the dynamic range exceeds 60 dB. As the frequency gets higher the dynamic range becomes smaller, finally going to -10 dB at 60 GHz. This is due to the 1/f roll-off of the spectrum of the step pulse generator; see Figure 2. Finally at very high frequencies, the spectral content is limited by the finite risetimes of both the pulse generator and the measurement oscilloscope. This is seen as a steeper slope beyond 50 GHz for the plots in Figure 8.

The next test to perform is a system 0 dB 'self-test' to determine repeatability and also again the maximum useful upper frequency limit. See Figure 9. For  $S_{21}$ , measure  $V_{out}$ , again without the DUT installed, but this time directly connect nodes 3 and 4, Figure 4, and again measure the input test signal. For  $S_{11}$ , measure  $V_{tdr}$ , but with the reference standard short circuit again attached at node 3, Figure 4. With these 'self-tests', the TDVNA should give the results of  $S_{21} = 0$  dB insertion loss and 0 ps group delay and also  $S_{11} = 0$  dB return loss. This test should be performed for both attenuation in dB and also phase (group delay). At very high frequencies, the plots of  $S_{21}$  and  $S_{11}$  will start to wildly deviate from 0 dB and 0 ps. What is happening is the available spectral energy in the test signal has dropped close to or below the system noise level. Thus, in equations 7 or 9 we have the situation of dividing (noise) / (noise) in which the result is indeterminate and could range from  $-\infty$  to  $+\infty$ .

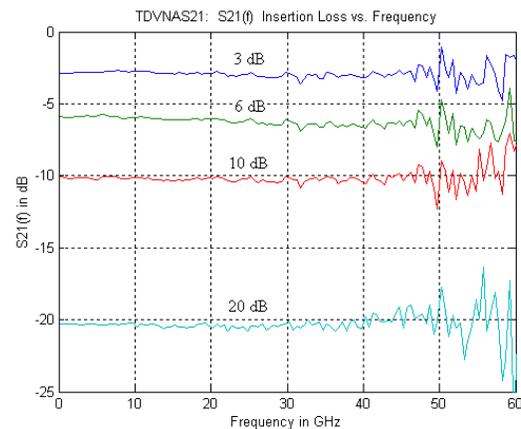


**Fig. 8** TDVNA system self-test of dynamic range. Test conditions were:  $N=4096$ ,  $T_w = 2$  ns, 256 averages.  $S_{21}$  (green),  $S_{11}$  (blue)



**Fig. 9** TDVNA system 0 dB self-test of repeatability and max. frequency. Test conditions were:  $N=4096$ ,  $T_w = 2$  ns, 256 averages.  $S_{21}$  (green),  $S_{11}$  (blue)

Additional TDVNA confidence checks that can be performed are to measure a set of precision, calibrated, coaxial attenuators. Short circuited attenuators can be used to provide various levels of known return loss. For example, a 3 dB attenuator will give -6 dB return loss when its far end is terminated in a short circuit. Figures 10-12 show such measurements on a set of PSPL model 5510K attenuators. The dc and low frequency attenuation values measured by the TDVNA agreed to within  $\pm 0.1$  dB of actual DC calibrations.



**Fig. 10** TDVNA  $S_{21}$  Insertion Loss measurements of precision PSPL model 5510K, 3 dB, 6 dB, 10 dB, and 20 dB, 2.95mm coax attenuators

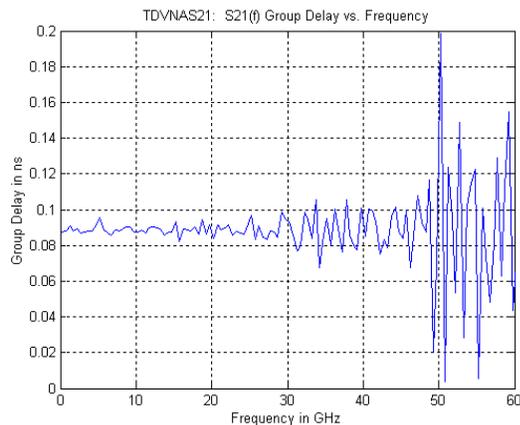


Fig. 11 TDVNA  $S_{21}$  Group Delay measurement of 6 dB, K attenuator

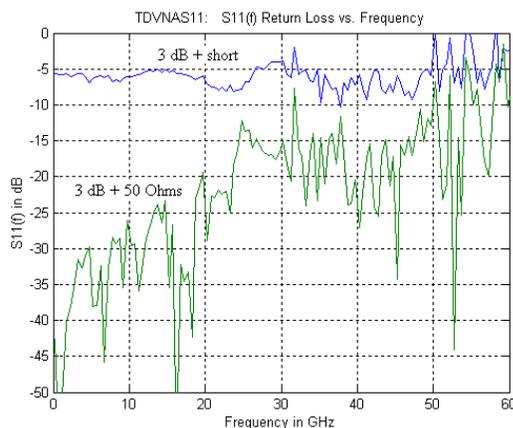


Fig. 12 TDVNA  $S_{11}$  Return Loss measurements of precision 3 dB K attenuator terminated in 50  $\Omega$  (green) and a short circuit (blue)

**MATLAB PROGRAMS** The most current versions of the actual *SpecAnalysis* and *TDVNA* MATLAB programs are available for download from the PSPL website at <http://www.picosecond.com>. They are written as simple MatLab \*.m (i.e., .txt) files. They can also be obtained directly from the author at: [jrandrews@picosecond.com](mailto:jrandrews@picosecond.com).

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