Picosecond Pulse Labs

Application Note AN-4

TDR, Step Response and "S" Parameter Measurements in the Time Domain

James R. Andrews, Ph.D., IEEE Fellow

Many different kinds of networks and devices can be characterized very easily by measurements in the time domain. They include both insertion and reflection measurements such as transition duration (10%-90% risetime), transient (step) response, impulse response, and Time Domain Reflectometry (TDR). Typical items to be tested include: amplifiers, coaxial cables, coaxial components, semiconductors, oscilloscopes, etc. An extensive list of references is included. Good review papers are ref. [1-3].

Frequency domain parameters can also be obtained from time domain measurements [4-7]. The frequency domain scattering "S" parameters $S11(j\omega)$, $S21(j\omega)$, $S22(j\omega)$, and $S12(j\omega)$ of a network are related to the time domain insertion waveforms through the Fourier Transformation. With computers and programmable, digital, sampling oscilloscopes, the time domain to frequency transformation (and vice versa) is easily accomplished. This in turn allows deconvolution methods [8-11] to be applied to calculate insertion impulse and step responses of driving point and transfer scattering parameters.

To perform these measurements in the picosecond time domain and GHz frequency domain requires extremely fast pulse generators and broadband sampling oscilloscopes. Picosecond Pulse Labs (PSPL) makes several pulse generators that are suitable. The two most popular PSPL generators are the Model TD-1107 and the Model 4050B. The TD-1107"D" is an ultra-fast tunnel diode pulser which produces a 230 mV step with less than 20 ps rise. The 4050B produces a 10 V step with a 45 ps rise. For impulse testing, Model 5210 is widely used. It is a passive network that converts these steps into impulses of 75 mV, 37 ps fwhm (50%) and 2.5 V, 50 ps fwhm, respectively.

Several excellent, wideband (DC to 12 or 20 GHz), digital sampling oscilloscopes are available for these measurements. They include: IWATSU model SAS-8130A, TEKTRONIX model 11802 and model 7854, and HEWLETT-PACKARD model 54120. These scopes are all programmable via the IEEE-488. Thus, they provide easy interfacing to a computer for FFTs and deconvolution.

TRANSFER INSERTION MEASUREMENTS

A typical application would be to measure the transition duration (risetime), Ta, of an amplifier using the PSPL Model TD-1107 as the input step pulse and a broadband sampling scope to observe the amplifier output step response. For Gaussian systems, the root-sum-of-squares equation is a good approximation for describing a system transition duration [12]. First measure Tm1 with the pulse generator directly attached to the sampling scope. Tg and Tss are the transition durations of the TD generator and the sampling scope, respectively.

$$Tm1 = (Tg^2 + Tss^2)^{1/2} \quad (1)$$

Next measure Tm2 with the amplifier inserted between the pulse generator and the sampling scope.

$$Tm2 = (Tg^2 + Ta^2 + Tss^2)^{1/2} \quad (2)$$

$$Ta = (Tm2^2 - Tm1^2)^{1/2} \quad (3)$$

Note that neither the generator nor the scope transition duration needs to be known to measure the amplifier transition duration.

The impulse response of an amplifier, etc., may be estimated from step response measurements. For Gaussian networks, the formula relating the impulse response 50% duration, FWHM, to the step response 10%-90% transition duration (risetime), Tr is:

$$FWHM = 0.92 \times Tr \quad (4)$$

The root-sum-of-squares equation (1) also holds for cascading the impulse responses of Gaussian networks.

The -3 dB bandwidth of amplifiers, etc., may also be estimated from the step response measurements. For Gaussian networks, the formula relating bandwidth (BW), and step response transition duration (Tr), is:

$$Tr(10\text{-}90\%) \times BW(-3\ dB) \approx 0.35 \quad (5)$$

The low frequency cutoff of AC-coupled amplifiers, etc., can also be measured using a step waveform such as from the TD-1107. The output will have a long-term sag in response to a flat step input. If the sag has a decaying exponential shape, then the low frequency cutoff, LF(-3 dB), is given by:

$$LF(-3\ dB) = 1 / (2 \times \pi \times \tau) \quad (6)$$

τ is the exponential time constant, i.e., the time at which the output has fallen to 37%. Sometimes it may not be possible to observe a complete exponential decay curve to measure τ. An observation of the percent sag in a pulse topline may be enough information to solve this equation for τ:

$$v(t) = V_o \times \exp(-t/\tau) \qquad (7)$$

"S(f)" PARAMETER MEASUREMENTS

Additional frequency domain, "S" parameter information can be obtained from time domain measurements [4-7] if a computer is connected to the sampling oscilloscope. The reference by Andrews [4] is recommended reading. A Fast Fourier Transform is used to transform time domain data to the frequency domain. For example, the frequency domain transfer function, H(f), or forward scattering parameter, S21(f), (gain and phase responses) of a network can be easily computed from step response measurements.

$$H(f) = S21(f) = FFT[Vm2(t)] / FFT[Vm1(t)] \qquad (8)$$

Vm1(t) is the measured transient test pulse with the pulse generator connected directly to the scope. Vm2(t) is the measured transient response with the network inserted between the generator and the scope. The actual impulse response of the network, h(t), can then be obtained using the inverse FFT.

$$h(t) = Inv.\ FFT\ [\ H(f)\ ] \qquad (9)$$

The corresponding actual step response is obtained by integrating (9) with respect to time. The operation (8) is called deconvolution. Care must be exercised in doing deconvolution. The presence of zeros or very small numbers in the denominator of equation 8 can cause the inverse transform of eqn. 10 to blow up. Uncorrelated noise in the measurement process and truncation within the computer can also cause the solution to blow up. The result of these errors is usually a very "dirty" computed step response with a lot of noise, spikes, ringing, and Gibb's phenomena. To avoid these problems, special deconvolution software filters are required. The reference by Nahman [9] is recommended reading along with several other references [8, 10, and 11].

TIME DOMAIN REFLECTOMETRY

Time Domain Reflectometry, or TDR, is an application for which the PSPL TD-1107 tunnel diode pulse generator is the ideal signal source. TDR is a measurement technique for evaluating the impedance quality of transmission line systems and/or components. TDR is basically a closed circuit RADAR. The list of references found in this application note include several that are recommended reading for additional details on TDR [13-18].

Figure 1 shows a basic TDR setup. The pulse generator is represented by its Thevenin equivalent circuit of an open circuit voltage source, Vg(t), and its output source impedance, Rg. The generator is connected through a feed-thru sampler to a reference transmission line of impedance R0 and length d. Normally Rg equals R0. The high-impedance feed-thru sampler of the oscilloscope is connected across the input terminals of the reference transmission line. The transmission line is terminated in an unknown impedance, Zt.

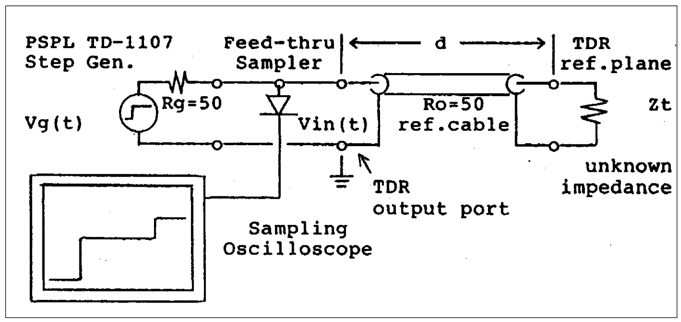

Figure 1: TDR System Configuration

Suitable feed-thru samplers include the IWATSU SH-4B, TEKTRONIX S-6, and HEWLETT-PACKARD 1430. Some engineers may wish to use terminated sampling heads for TDR. Examples are the TEKTRONIX S-4, SD-26 and HEWLETT-PACKARD 54121A. These are not feed-thru samplers, but are internally terminated in 50 Ohms. They present a problem for the feed-thru arrangement of Figure 1. Figure 2 shows how to use these samplers for TDR. This requires the use of either a 10X Coaxial Signal Probe, such as the PSPL Model 5520A, or a matched-impedance, 6 dB power divider, such as the PSPL Model 5330.

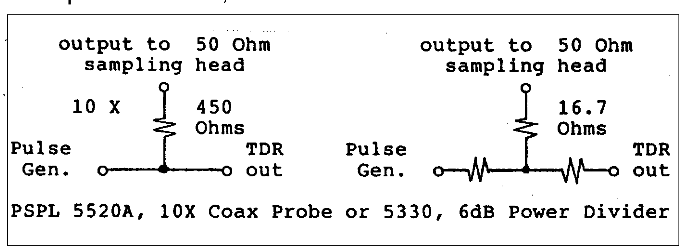

Figure 2: TDR Arrangement for 50 Ohm Internally-Terminated Sampling Heads

In TDR, the pulse generator sends a pulse through the sampler into the reference transmission line. The pulse propagates through the line at a velocity, vp, and arrives at the far end after a time TD.

$$TD = d / vp \quad (10) \qquad vp = c / (\varepsilon)^{1/2} \quad (11)$$

Where c is the speed of light (3×10^{10} cm/sec.) and ε is the relative dielectric constant of the transmission line. If the load impedance matches the line impedance, Zt = Ro, then the TDR pulse is perfectly absorbed. However, if Zt is not equal to Ro, some of the incident pulse energy will be reflected back to the left toward the generator. This reflected pulse will arrive at the TDR output port at t = 2TD. The feed-thru oscilloscope allows us to visually see the total waveform, Vin (t). Vin (t) is the algebraic sum of the pulse generator pulse and any returning echoes. Thus, an examination of the time delay and wave shape of the echoes present on Vin(t) allows us to determine the location and nature of discontinuities within the line and/or mismatched terminations to the line.

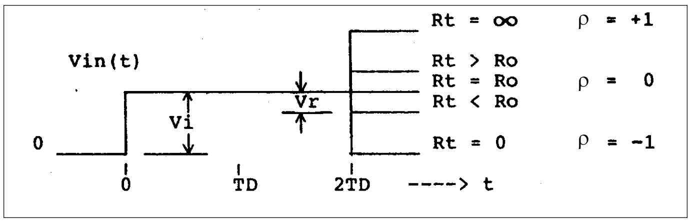

Figure 3: TDR Waveforms with Resistive Terminations

Figure 3 shows the TDR waveforms observed for various resistive terminations, Rt. For this and the rest of the TDR discussion, we will assume that the generator produces a step function pulse. If Rt = Ro, then no reflection occurs and the TDR display on the scope is a flat line. If Rt is greater than Ro, then a positive step is observed. For Rt < Ro, a negative step is observed. The actual value of Rt may be calculated from the size of these steps. The amplitude of the incident step (Vi) and the reflected pulse (Vr) are measured (Figure 2). The reflection coefficient, ρ, is defined as:

$$\rho = Vr / Vi \qquad (12)$$

Note that Vr may have either a positive or negative value and likewise for ρ. The example shown in Figure 3 is for a negative reflection. Transmission line analysis shows that ρ is also given by:

$$\rho = (Rt - Ro) / (Rt + Ro) \qquad (13)$$

Rearranging terms in (13) allows us to solve for Rt.

$$Rt = Ro \times (1 + \rho) / (1 - \rho) \qquad (14)$$

For the matched case, Rt = R0 and $\rho = 0$. For an open circuit, $\rho = +1$. For a short circuit, Rt = 0 and $\rho = -1$. Equations 12-14 hold for both pure resistive terminations and connections to other transmission lines of different characteristic impedances.

Reactive components can also be measured using TDR. Figure 4 shows the TDR waveforms obtained for simple inductors and capacitors. The inductor initially appears as an open circuit to the fast rising edge of the TDR step pulse (i.e., the high frequencies). Thus, it initially gives a ρ of +1. Later in time, the inductor appears as a short circuit to the flat top of the TDR step pulse (i.e., the DC portion). Therefore, the final TDR value is a ρ of -1. The capacitor performs exactly opposite. The L or C value may be determined by measuring the exponential time constant, τ, of the TDR response.

$$L = Ro \times \tau \qquad (15) \qquad\qquad C = \tau / Ro \qquad (16)$$

Figure 5 shows a very common situation encountered when dealing with coaxial cables. These are the cases of either a series inductance (L) or a shunt capacitance (C) within the coax. This could be caused by a poor connector, for example. Again, by measuring the time constant τ, L or C may be determined.

$$L = (2 \times Ro) \times \tau \qquad (17) \qquad\qquad C = \tau / (Ro/2) \qquad (18)$$

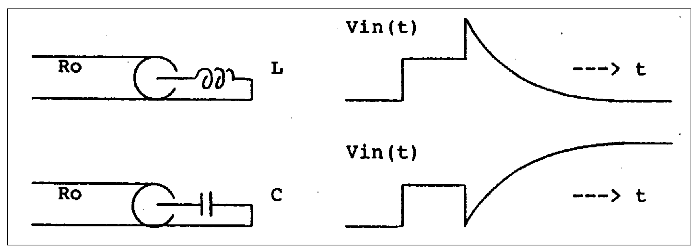

Figure 4: TDR of L and C Terminations

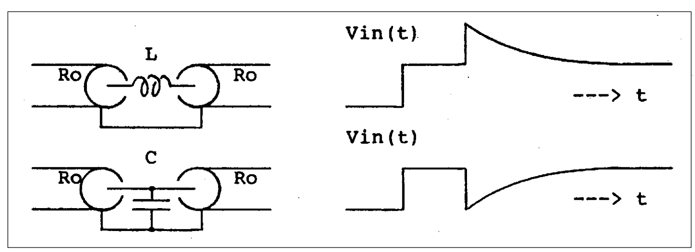

Figure 5: TDR of Series L and Shunt Within a Cable

The TDR waveforms shown in Figures 4 and 5 are for the case when the TDR step waveform is ideal with a 0 ps risetime. With a finite-risetime pulse generator and sampling oscilloscope, these waveforms will no longer have sharp corners, but will have smooth rounded corners. For “Large” inductors or capacitors, their time constants will be much greater than the system risetime, Tr. In their cases, the TDR displays will still be as shown in Figures 4 and 5. No visible reflections will occur for “Tiny” inductors or capacitors with time constants much less than the TDR system risetime, Tr. Thus, they can not be measured.

Figure 6 shows the TDR displays that will occur for "Small" inductors and capacitors. This is the situation when the time constant associated with the reactive component is of the same order of magnitude as the TDR system risetime, Tr. These "Small" inductors or capacitors can be measured even though their TDR responses are not the same as Figures 4 and 5. First measure Tr, Vi and Vpk as shown in Figure 6. Equations 19 and 20 are approximate formulas used to calculate L and C [7-9].

$$L = (2 \times Ro) \times Tr \times [Vpk / (0.8 \times Vi)] \qquad (19)$$

$$C = (2 / Ro) \times Tr \times [Vpk / (0.8 \times Vi)] \qquad (20)$$

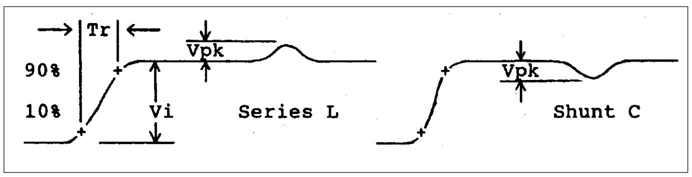

Figure 6: TDR of "Small" L and C Reactances in Coax

For impedance discontinuities located very close to the TDR output port, the risetime of the displayed output step, Figure 6, may be used as Tr. However, if a long cable is present between the TDR output and the reference plane at which the measurements are to be made, then the effective risetime will be slower than the input risetime due to the cable pulse response. In this case, the system should first be calibrated by attaching a short circuit at the reference plane and measuring the falltime, Tf, of the reflection from the short, Figure 7. Use the short circuit falltime in place of Tr in equations 19 and 20.

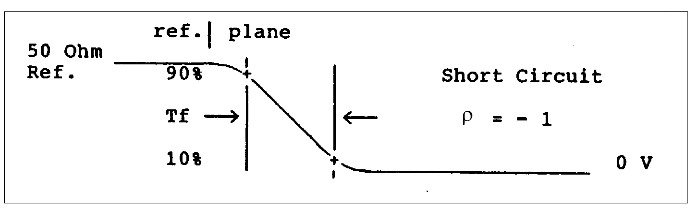

Figure 7: TDR System Calibration Short Circuit at the end of the 50 Ohm Coaxial Reference Line

Figure 8 illustrates the TDR display for two closely-spaced discontinuities. The minimum temporal resolution is the system risetime, Tr. The minimum spatial resolution, X(min), is given by:

$$X(min) = 0.5 \times c \times Tr / (\varepsilon)^{1/2} \qquad (21)$$

As an example, with a 25 ps generator and a 25 ps scope, the TDR system risetime would be 35 ps. The resultant minimum spatial resolution in air dielectric would be 5 mm.

If a computer is available, considerably more information can be extracted from TDR waveforms [4-7]. The frequency domain, scattering parameter S11(f) can be computed using FFTs.

$$S11(f) = FFT[Vtdr(t)] / FFT[-Vsc(t)] \qquad (22)$$

Vsc(t) is the measured reflection waveform from a short circuit, Figure 7. Vtdr(t) is the measured TDR waveform of the device under test.

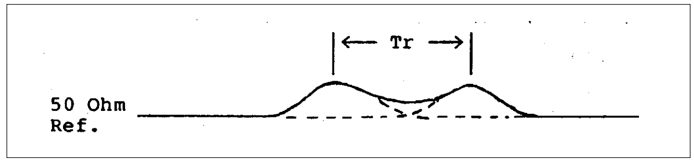

Figure 8: Minimum Temporal/Spatial Resolution

Using deconvolution techniques, it is also possible with a computer to correct a TDR waveform, such as Figure 6, to give a TDR waveform that would result if a near-perfect step function had been used as the test signal. All TDRs show rather "dirty" TDR reference baselines that are corrupted by small bumps and wiggles that are present on the step generator and minor reflections within the sampler. Deconvolution effectively removes most of these effects. It can also give an effective increase in bandwidth and reduction in the TDR system risetime and hence its spatial resolution. Software signal processing can also slow down the TDR's system risetime if desired to show what reflections might occur in a transmission system when slower risetime pulses are used.

DIFFERENTIAL TDR

The discussion up to now has dealt with single-ended coaxial cable TDR. Differential TDR is important for performing the same class of measurements on balanced transmission lines on p.c. boards and twisted-pair cables. This requires a balanced pulse generator and sampler. In the picosecond domain these do not exist. However, a differential TDR can be configured using unbalanced coaxial components, Figure 9. A PSPL Model 5320A Differential Pulse Splitter is used to split the TD-1107 step pulse into two 110 mV pulses of opposite polarities. Two feed-thru sampling heads are now required. The signals from the sampling heads can then be added algebraically using the scope main frame (CH A minus CH B) mode to give the differential TDR signal.

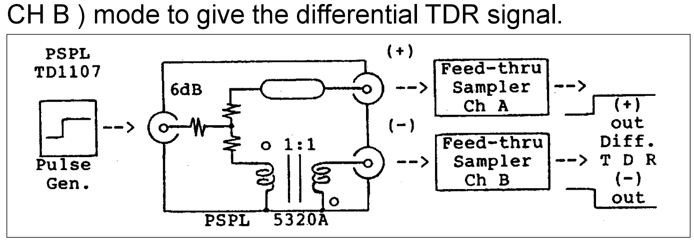

Figure 9: Differential TDR Using the PSPL Model 5320A Differential Pulse Splitter

REFERENCES

[1] N.S. Nahman," Picosecond-Domain Waveform Measurement: Status and Future Directions", IEEE Trans Inst & Meas, vol. IM32, no. 1, Mar. 1983, pp. 117-124.
[2] R. Lawton, S. Riad, and J. Andrews," Pulse and Time-Domain Measurements", Proc. of IEEE, vol. 74, no. 1, Jan., 1986, pp. 77-81.
[3] E.K. Miller, Editor, TIME-DOMAIN MEASUREMENTS IN ELECTROMAGNETICS, Van Nostrand Reinhold Co., New York, 1986, 536 pages.
[4] J.R. Andrews, "Automatic Network Measurements in the Time Domain", Proceedings of the IEEE, vol. 66, no. 4, April, 1978, pp. 414-423.
[5] J. R. Andrews and W. Gans, "Time Domain Automatic Network Analyzer", L'Onde Electrique, Paris, vol. 55, no. 10, Dec., 1975. pp. 569-574.
[6] W.L. Gans & J.R. Andrews," Time Domain Automatic Network Analyzer for Measurement of RF & Microwave Components", NBS Tech. Note 672, NBS, Boulder, Colo., Sept., 1975.
[7] R. Cole, "Time-Domain Spectroscopy of Dielectric Materials", IEEE Trans Inst & Meas., vol. IM-25, no. 4, Dec., 1976, pp. 371-375.
[8] S. Riad," The Deconvolution Problem: An Overview", Proc. of IEEE, vol. 74, no. 1, Jan., 1986, pp. 82-85.
[9] N.S. Nahman, "Software Correction of Pulse Measurement Data", HIGH SPEED ELECTRICAL & OPTICAL PULSE MEASUREMENTS, J.E. Thompson, editor, Martinus Wijhoff Publisher, Boston, Mass., 1986, pp. 351-417.
[10] N.S. Nahman and M.E. Guillaume, "Deconvolution of Time Domain Waveforms in the Presence of Noise", NBS Tech. Note 1047, NBS, Boulder, Colo., Oct., 1981.
[11] B. Parruck and S. Riad,"An Optimization Criterion for Iterative Deconvolution", IEEE Trans Inst & Meas., vol. IM32, no. 1, Mar., 1983, pp. 137-140.
[12] R.D. Thornton et. al., Chp. 8 "Interrelations Between Frequency Domain, Time Domain & Circuit Parameters, MULTISTAGE TRANSISTOR CIRCUITS, SEEC, vol. 5, John Wiley & Sons, New York, 1965.
[13] J. Strickland, A. Zimmerman, G. Long, & G.Frye,"Time-Domain Reflectrometry Measurements", Tektronix, Beaverton, Oregon, 1970, p/n 062-1244-00.
[14] "Time Domain Reflectrometry" Application Note No. 62, Hewlett-Packard, Palo Alto, Calif., 1964. Also in HP Journal, vol.15, no.6, Feb.1964, pp.1-8. Reprinted in INVENTIONS of OPPORTUNITY, Hewlett-Packard, Palo Alto, Calif. 1983.
[15] N.S. Nahman et. al., "Reference Waveform Generation Using Debye Dielectric Dispersion", NBSIR 73-304, Dec., 1972, NBS, Boulder, Colo., (avail. from NTIS Springfield, VA 22161). See pp. 61--85 for a detailed derivation of the complete TDR equations.
[16] J. Millman and H. Taub, PULSE DIGITAL & SWITCHING WAVEFORMS, chp.3, McGraw-Hill, New York, 1965.
[17] B.J. Elliott, "High Sensitivity Picosecond Time Domain Reflectrometry", IEEE Trans. Inst. & Meas., vol. IM-25, no. 4, Dec., 1976, pp. 376-379.
[18] S. Riad & N. Nahman, "Application of the Homomorphic Deconvolution for the Separation of TDR Signals Occurring in Overlapping Time Windows", IEEE Trans. Inst.& Meas., vol. IM-25, no.4,Dec.,1976, pp. 388-391.
[19] "TDR Fundamentals", HP Appl. Note 62, Hewlett-Packard, Palo Alto, CA, April 1988. (note app. note 62 is same number as used by HP for ref.14, 1964)
[20] "Improving Time Domain Network Analysis Measurements", HP Application Note 62-1, Hewlett-Packard, Palo Alto, Calif., April 1988.